

Anamorphic Signatures

Secrecy From a Dictator Who Only Permits Authentication!

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Joint work with Miroslaw Kutylowski, Duong Hieu Phan, Moti Yung,
Marcin Zawada

Privacy as a Human Right

UDHR, Article 12: (1948)

*No one shall be subjected to arbitrary interference with his privacy, family, home or **correspondence**,...*

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End to End Encryption

- Cryptography has been very successful in providing tools for encrypting communication
 - ▶ The Signal protocol and app



The receiver-privacy assumption

Encryption guarantees message confidentiality only with respect to parties that do not have access to the receiver's private key

The receiver-privacy assumption

The receiver keeps his secret key in a private location

Receiver privacy

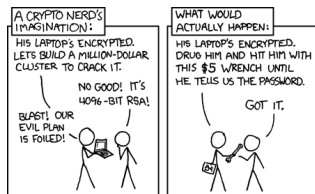
- Realistic for “normal” settings
- No wonder Encryption has been developed under these assumptions
 - ▶ with no explicit mention

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Receiver privacy

- Realistic for “normal” settings
- No wonder Encryption has been developed under these assumptions
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- In a dictatorship, instead
 - ▶ No receiver privacy: citizens might be “invited” to surrender their private keys



Democracies attempt to regulate encryption

The Clipper Chip

Presently, anyone can obtain encryption devices for voice or data transmissions. [...] if criminals can use advanced encryption technology in their transmissions, electronic surveillance techniques could be rendered useless because of law enforcement's inability to decode the message.

Howard S. Dakoff

The Clipper Chip Proposal, J. Marshall L. Rev., 29, 1996.

Ban of E2E encryption

In our country, do we want to allow a means of communication between people which even in extremis, with a signed warrant from the Home Secretary personally, that we cannot read?

David Cameron
UK Prime Minister
January 2015

Crypto Wars

Arguments against restricting encryptions:

- *the bad guys can utilize other encryption systems*
- *all other encryption schemes must be declared illegal*
 - ▶ what qualifies as an encryption scheme? e.g., *chaffing and winnowing*
- *it creates a natural weak system security point*
 - ▶ Frankel and Yung showed how to frame legitimate users in the Clipper Chip [Crypto 95]

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indirect and non-technical

A Dictator's Paradise

A thought experiment

- Let us have it the Dictator's way
 - ▶ Encryption still allowed
 - ▶ Users must surrender the secret keys associated with their public keys
- Did the dictator achieve their goal?
 - ▶ Access to all communication

Enter Anamorphic Encryption

[P, Phan, Yung – Eurocrypt 22]

The anamorphic approach

- A ciphertext is associated with **two** secret keys **sk**, **dkey**
- share **dkey** with your friend
- A ciphertext **ct** carries two plaintexts m_0, m_1 , one for each key
 - ▶ $m_0 = \text{Dec}(ct, sk)$, the regular decryption algorithm
 - ▶ $m_1 = \text{aDec}(ct, dkey)$, the **anamorphic** decryption algorithm
- ...and there is **no** second key
 - ▶ at least, that's what the dictator thinks
 - ▶ when dictator asks for keys, give him **sk** because *there is only one key...*

The anamorphic thesis

The anamorphic thesis

Regulating/crippling encryption is technically **futile**

- **Because** Anamorphic Encryption exists
 - ▶ Feasibility of Anamorphic Encryption [P, Phan, Yung – EC22]
 - ★ The Naor-Yung CCA encryption scheme
- **Because** Anamorphic Encryption is being used
 - ▶ Prevalence of Anamorphic Encryption [Kutilowski, P, Phan, Yung, Zawada – PETS 23]
 - ★ Paillier, RSA-OAEP, Goldwasser-Micali
 - ★ ElGamal, Cramer-Shoup, Smooth Projective Hash Functions

Resistance is futile



A Frequent Objection

*The dictator can hit you with the wrench until you surrender the **second** key*

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*Rather to tell democracies how low they must go to **effectively** control private communication*

and then ask if it is worth it

Futile, you said?

Encryption is declared illegal

- the dictator mandates that all communication happens through a central hub
- messages can only be digitally signed
 - ▶ so that we know whom we are talking to

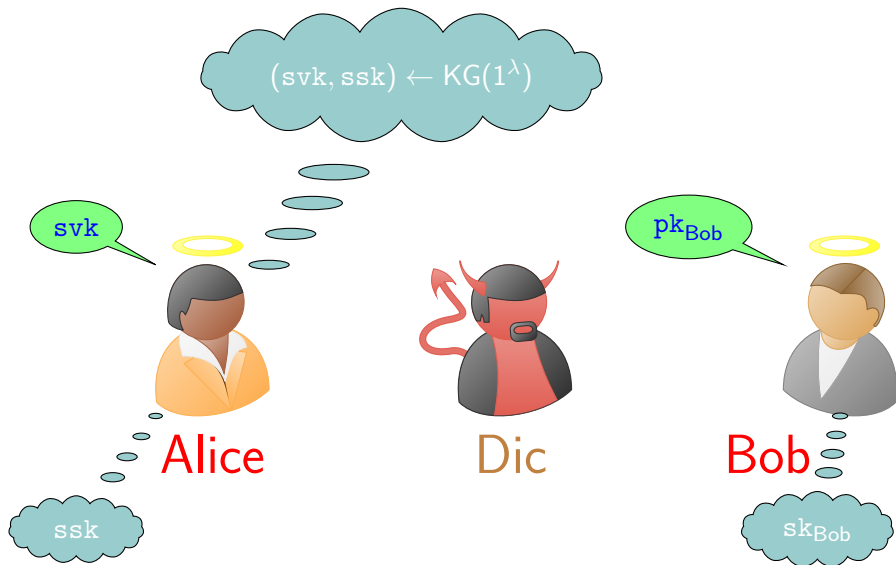
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Do not annoy your dictator!

The new dictatorial setting



The new dictatorial setting

msg = "Glory to Dic" \rightarrow Bob

svk



Alice

ssk



Dic

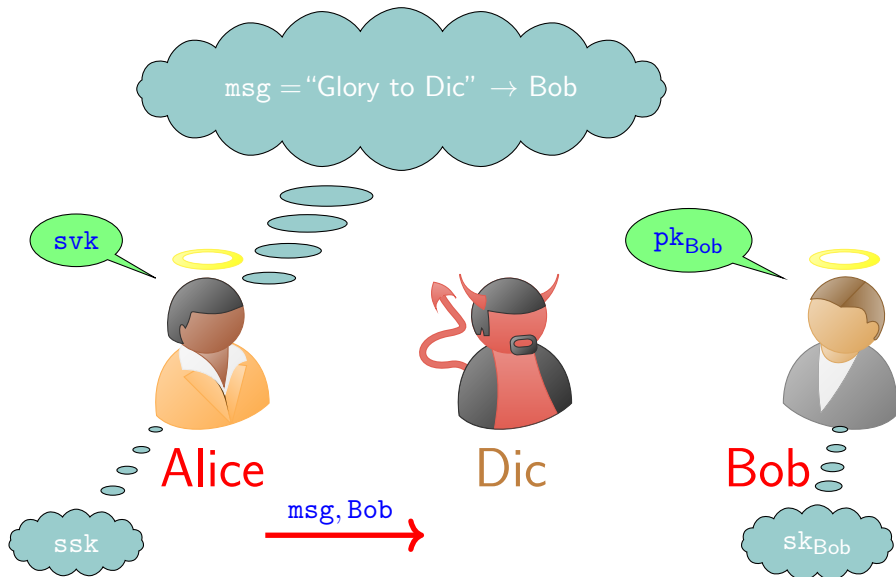
pk_{Bob}



Bob

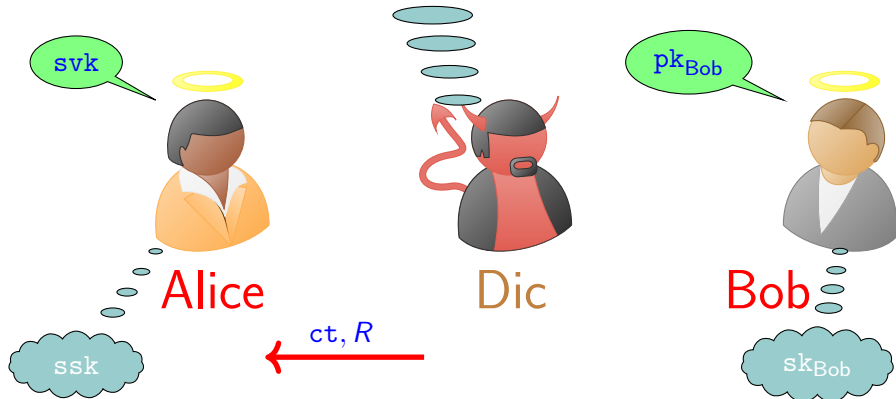
sk_{Bob}

The new dictatorial setting

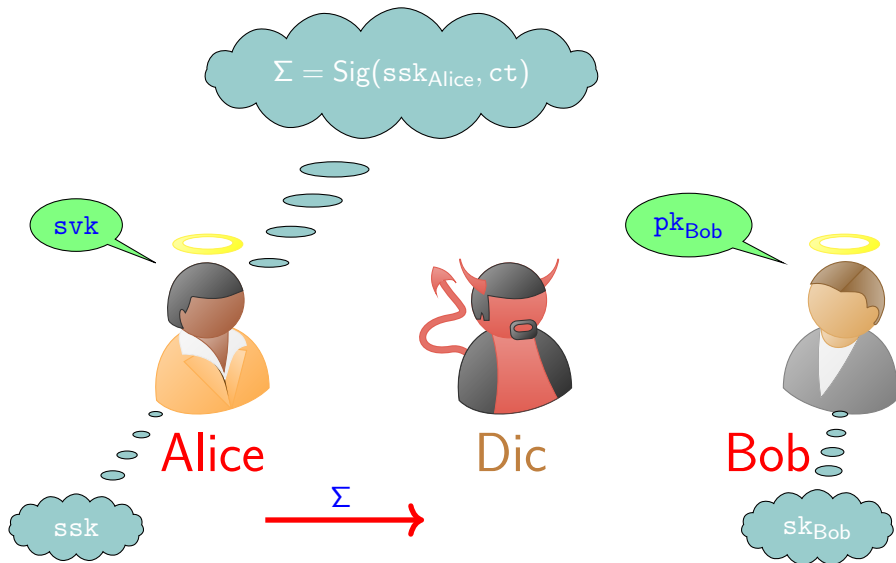


The new dictatorial setting

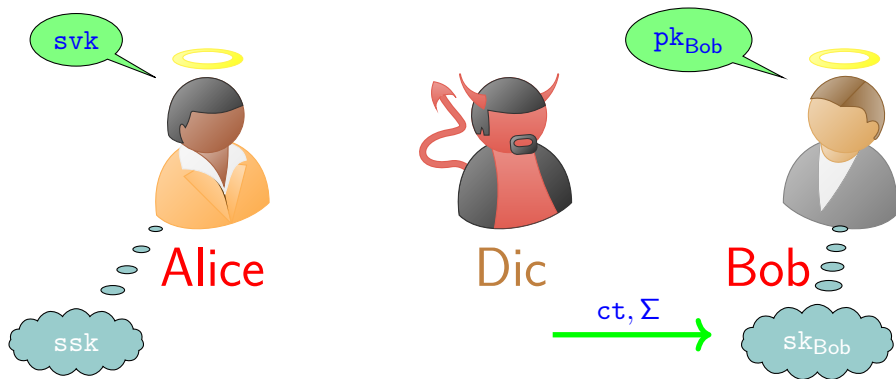
$$ct = \text{Enc}(pk_{\text{Bob}}, \text{msg}; R)$$



The new dictatorial setting



The new dictatorial setting



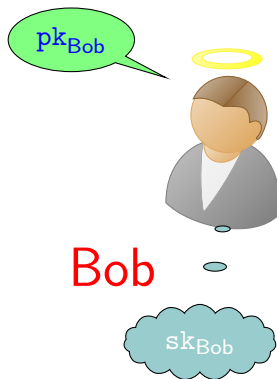
Dictator's thinking

- Every user has a private channel to the Dic
- Every user has a public and secret encryption key
- Every user has a verification and signing key
- Dic is the only allowed to use encryption on a public channel

The new anamorphic setting



Alice



Bob

The new anamorphic setting

$$(svk, ssk, dkey) \leftarrow aKG(1^\lambda)$$

svk



Alice

ssk, dkey

dkey

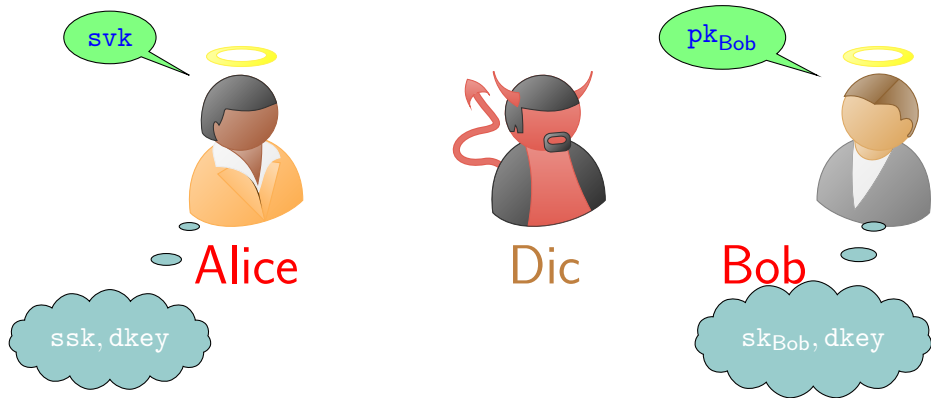
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Dic

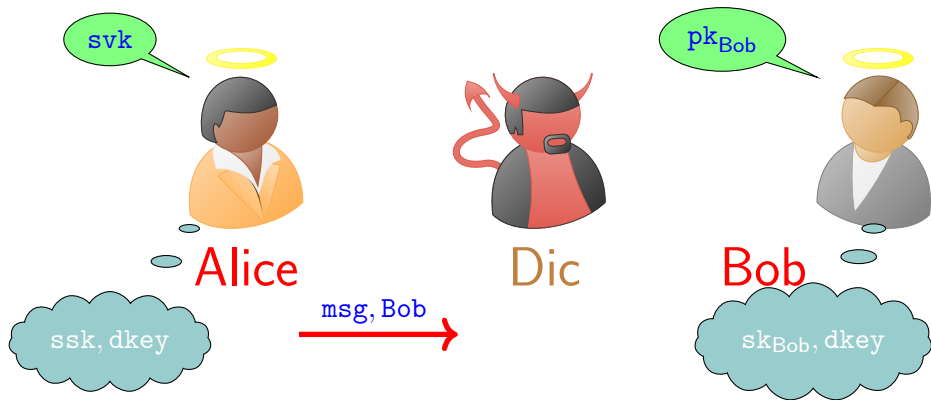
pk_{Bob}



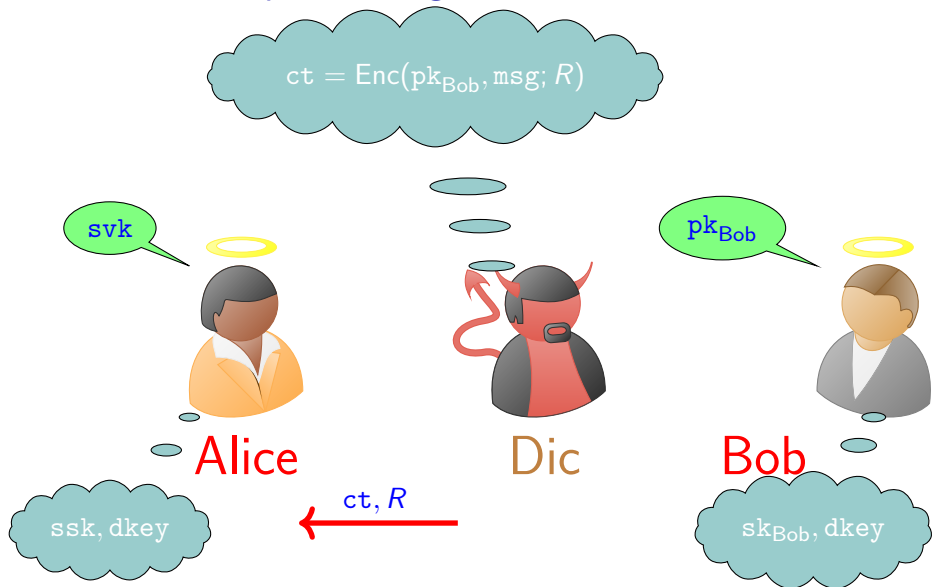
Bob

sk_{Bob}, dkey

The new anamorphic setting



The new anamorphic setting



The new anamorphic setting

amsg = "Fight Dic" → Bob

svk



Alice

ssk, dkey



Dic

pk_{Bob}

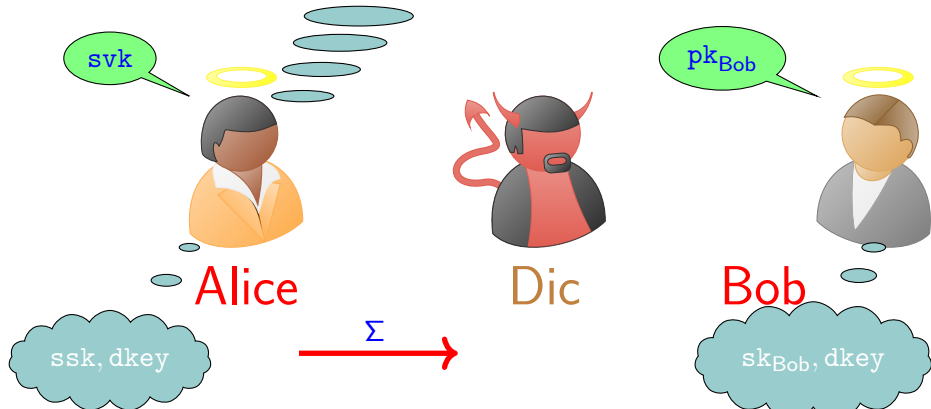


Bob

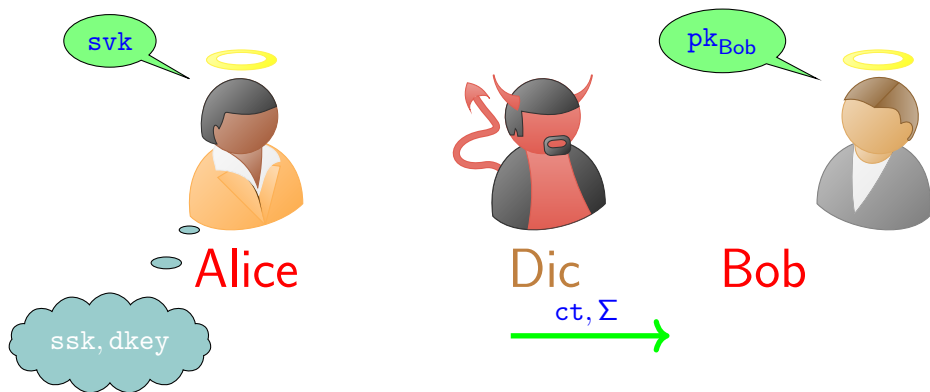
sk_{Bob}, dkey

The new anamorphic setting

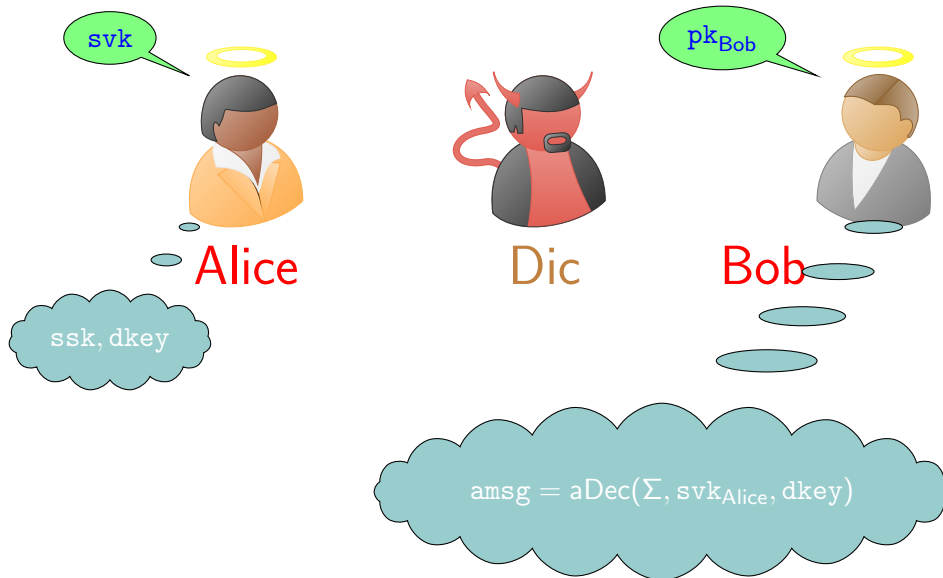
$$\Sigma = \text{aSig}(\text{ssk}_{\text{Alice}}, \text{ct}, \text{amsg}, \text{dkey})$$



The new anamorphic setting



The new anamorphic setting



Signature Schemes

- the *key-generation* algorithm $\text{KG}(1^\lambda)$
 - ▶ (svk, ssk) , a public *verification* key and secret *signing* key;
- the *signing* algorithm $\text{Sig}(\text{msg}, \text{ssk})$
 - ▶ *signature* Σ ;
- the *verification* algorithm $\text{Verify}(\Sigma, \text{msg}, \text{svk})$
 - ▶ accepts or rejects Σ as a signature of msg .

Anamorphic Triplet

- the *anamorphic key-generation* algorithm $\text{aKG}(1^\lambda)$
 - ▶ $(\text{svk}, \text{ssk}, \text{dkey})$, a public *verification* key, a secret *signing* key, and a *double* key;
- the *anamorphic signing* algorithm $\text{aSig}(\text{msg}, \text{amsg}, \text{ssk}, \text{dkey})$
 - ▶ *anamorphic signature* Σ ;
- the *anamorphic decryption* algorithm $\text{aDec}(\Sigma, \text{svk}, \text{dkey})$
 - ▶ amsg .

Security Notion for Anamorphic Signatures

RealG_{S, D}(λ)

- 1 (svk, ssk) \leftarrow KG(1^λ);
- 2 return $\mathcal{D}^{\text{Os}(\cdot, \cdot, \text{ssk})}(\text{svk}, \text{ssk})$, where
 $\text{Os}(\text{msg}, \text{amsg}, \text{ssk}) = \text{Sig}(\text{msg}, \text{ssk})$.

AnamorphicG_{T, D}(λ)

- 1 (asvk, assk, dkey) \leftarrow aKG(1^λ);
- 2 return $\mathcal{D}^{\text{Oa}(\cdot, \cdot, \text{assk}, \text{dkey})}(\text{asvk}, \text{assk})$, where
 $\text{Oa}(\text{msg}, \text{amsg}, \text{assk}, \text{dkey}) = \text{aSig}(\text{msg}, \text{amsg}, \text{assk}, \text{dkey})$.

Two good points raised by the audience

- Going back to our motivating scenario:
 - ▶ msg is the ciphertext ct produced by \mathcal{D} that carries the innocent looking message “Glory to Dic”.
 - ▶ $amsg$ is the message “Fight Dic” that Alice wants to secretly send to Bob.

To get a stonger notion of security, we require indistinguishability to hold even if \mathcal{D} picks the two messages

- If \mathcal{D} has access to ssk , what is the point of having Alice sign the ciphertext?
 - ▶ in the motivating scenario (see previous slides), \mathcal{D} does not have access to ssk
 - ▶ still we require indistinguishability to hold in case \mathcal{D} has some doubt and requests Alice’s signing key

Anamorphic Channels

- **dkey** can be used to establish an **anamorphic channel** between signers and verifiers that have access to dkey
- The channel can be **One-to-Many**
 - ▶ **dkey does not give you the ability to sign**
 - ▶ **only the signer can send anamorphic messages**
- The channel can be **Many-to-Many**
 - ▶ **dkey does give you the ability to sign**
 - ▶ **everybody is a signer and can send anamorphic messages**

Many-To-Many

$$(svk, ssk, dkey) \leftarrow aSig(1^\lambda)$$

Alice



Many-To-Many

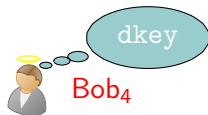
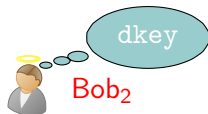
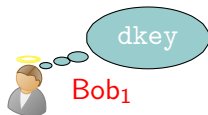


Many-To-Many

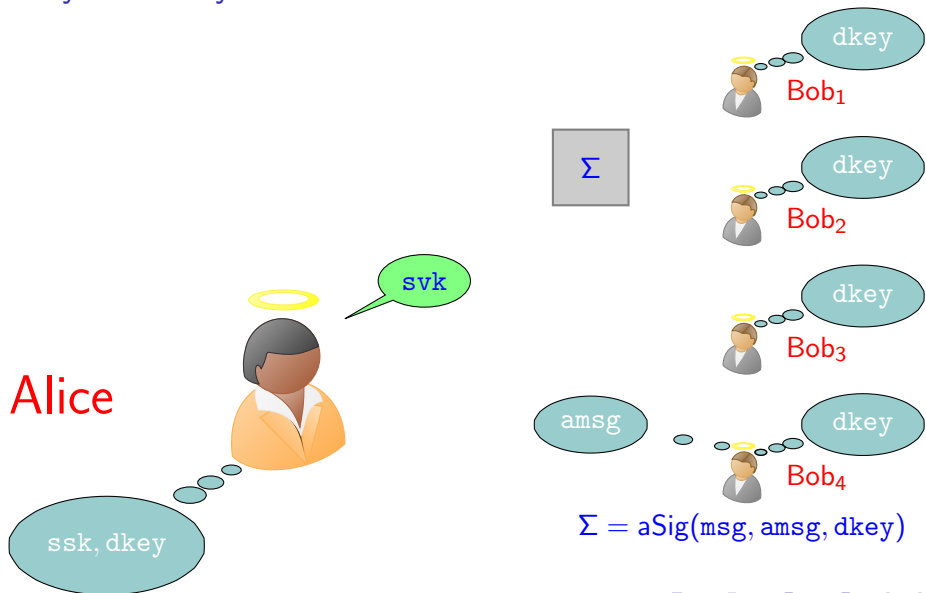


Many-To-Many

Alice



Many-To-Many



Many-To-Many

$$\text{amsg} = \text{aDec}(\Sigma, \text{svk}, \text{dkey})$$

Alice

amsg

svk

ssk, dkey

Σ

dkey

Bob₁

dkey

amsg

Bob₂

dkey

amsg

Bob₃

dkey

amsg

Bob₄

One-To-Many

$$(svk, ssk, dkey) \leftarrow aSig(1^\lambda)$$

Alice



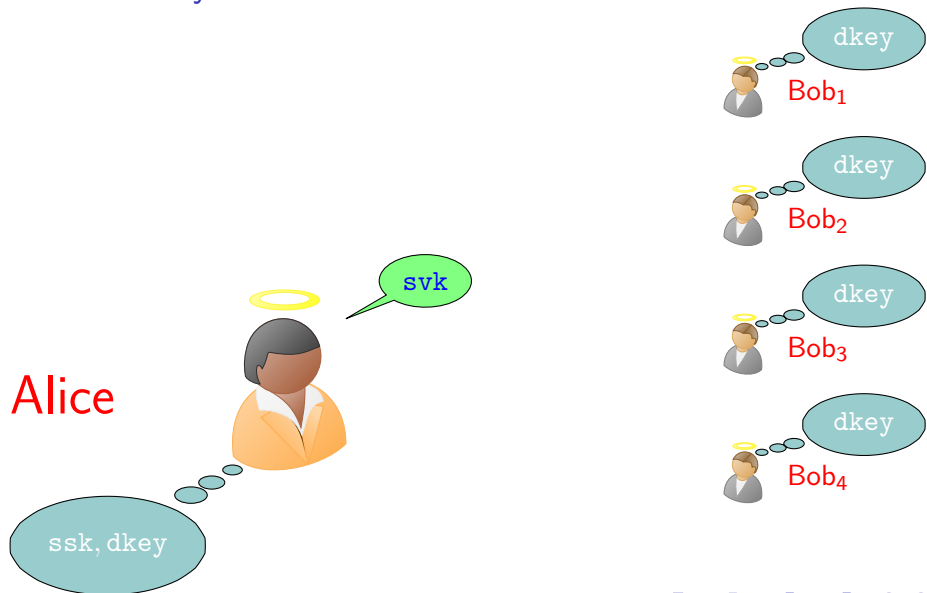
One-To-Many



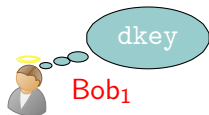
One-To-Many



One-To-Many

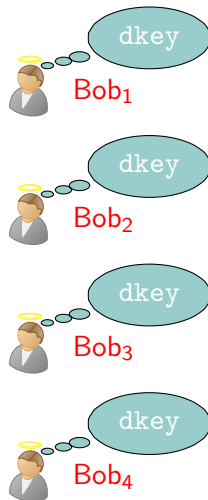


One-To-Many

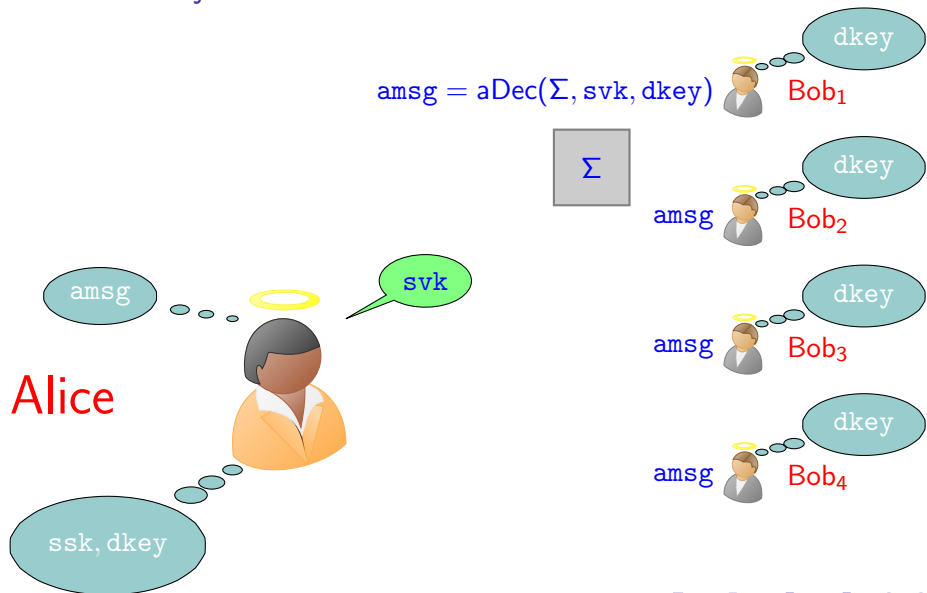


One-To-Many

$$\Sigma = \text{aSig}(\text{msg}, \text{amsig}, \text{ssk}, \text{dkey})$$



One-To-Many



One-to-Many Anamorphic Signatures

$D_{\text{sig}}G_{S,T}^A(\lambda)$

- 1 $(\text{asvk}, \text{assk}, \text{dkey}) \leftarrow \text{aKG}(1^\lambda)$;
- 2 $(\text{msg}, \Sigma) \leftarrow \mathcal{A}^{\text{Os}(\cdot, \text{assk})}(\text{asvk}, \text{dkey})$,
where $\text{Os}(m, \text{assk}) = (m, \text{Sig}(m, \text{assk}))$;
- 3 if $\text{Verify}(\Sigma, \text{msg}, \text{asvk}) = 1$ and (msg, Σ) has not been returned by Os then return 1; else return 0.

A General Technique

- **dkey** includes the key K of a symmetric encryption scheme
- A two step procedure
 - ▶ identify *extractable* randomness from the signature
 - ▶ replace randomness with **ciphertext** encrypted using K

ciphertexts must be indistinguishable from random

Symmetric Encryption with PseudoRandom Ciphertexts

prEnc returns $\ell(\lambda)$ -bit ciphertexts for encrypting $n(\ell)$ -bit messages with a key with security parameter λ

$\text{PRCtG}_{\text{prE}, \mathcal{A}}^\beta(\lambda)$

- 1 Set $K \leftarrow \text{prKG}(1^\lambda)$
- 2 Return $\mathcal{A}^{\text{OPr}^\beta(K, \cdot)}()$, where, for $n(\lambda)$ -bit plaintext msg ,
 $\text{OPr}^0(K, \text{msg})$ returns a randomly selected $\ell(\lambda)$ -bit string;
 $\text{OPr}^1(K, \text{msg}) = \text{prEnc}(K, \text{msg})$.

The Boneh-Boyen signature scheme

- The **Key Generation** algorithm $\text{bbKG}(1^\lambda)$
 - ▶ $(G_1, G_2, G_T, e, p) \leftarrow \mathcal{G}(1^\lambda)$
 - ▶ **Generators** $g_1 \in G_1, g_2 \in G_2$
 - ▶ $x, y \leftarrow \mathbb{Z}_p$
 - ▶ $z = e(g_1, g_2), u = g_2^x, v = g_2^y$.
 - ▶ $\text{svk} = (g_1, g_2, u, v, z)$ and $\text{ssk} = (g_1, x, y)$.
- The **Signing** algorithm $\text{bbSig}(\text{ssk} = (g_1, x, y), \text{msg} \in \mathbb{Z}_p)$
 - ▶ randomly selects $r \leftarrow \mathbb{Z}_p$.
 - ▶ If $r = -(x + \text{msg})/y$ then \perp .
 - ▶ return $(r, \sigma = g_1^{1/(x+\text{msg}+yr)})$.
- The **Verification** algorithm $\text{bbVerify}(\Sigma = (r, \sigma))$
 - ▶ check

$$e(\sigma, u \cdot g_2^{\text{msg}} \cdot v^r) = z.$$

Anamorphic Triplet for BB

- The *anamorphic key generation* algorithm $\text{abbKG}(1^\lambda)$
 - ▶ $(\text{svk}, \text{ssk}) \leftarrow \text{bbKG}(1^\lambda)$
 - ▶ $K \leftarrow \text{prKG}(1^\lambda)$.
 - ▶ return
 - ★ *anamorphic verification key* $\text{asvk} := \text{svk}$,
 - ★ *anamorphic signing key* $\text{assk} := \text{ssk}$,
 - ★ *double key* $\text{dkey} := K$
- The *anamorphic signing* algorithm $\text{abbSig}(\text{msg}, \text{amsg}, \text{ssk}, \text{dkey})$
 - ▶ $\text{act} = \text{prEnc}(K, \text{amsg})$
 - ▶ $r = \text{act}$ and if $r = -(x + m)/y$ then \perp
 - ▶ return $\text{a}\Sigma = (r, \sigma = g_1^{1/(x+m+yr)})$.
- The *anamorphic decryption* algorithm $\text{aDec}(\text{a}\Sigma = (r, \sigma), \text{dkey} = K)$
 - ▶ return $\text{amsg} = \text{prDec}(K, r)$.

The Fiat-Shamir Heuristics

The Schnorr Signature Scheme

- The *Key Generation* algorithm $\text{ScKG}(1^\lambda)$

- ▶ \mathbb{G} , a cyclic group of prime order q
- ▶ a generator $g \in \mathbb{G}$
- ▶ hash function $H : \{0, 1\}^* \times \mathbb{G} \rightarrow \mathbb{Z}_q$.
- ▶ $x \leftarrow \mathbb{Z}_q$ and sets $y = g^x$

$\text{ssk} := (\mathbb{G}, g, H, x)$ and $\text{svk} := (\mathbb{G}, g, H, y)$.

- The *Signing* algorithm $\text{ScSig}(\text{ssk}, \text{msg})$

- ▶ $\kappa \leftarrow \mathbb{Z}_q$,
- ▶ $r = g^\kappa$, $c = H(\text{msg}, r)$ and $s = \kappa + c \cdot x$
- ▶ Return $\Sigma = (r, s)$.

- The *Verification* algorithm $\text{ScVerify}(\Sigma, \text{msg}, \text{svk})$

- ▶ checks that

$$r = g^s \cdot y^{-H(\text{msg}, r)}$$

Anamorphic Schnorr

- The *Signing* algorithm $\text{ScSig}(\text{ssk}, \text{msg})$
 - ▶ $\kappa \leftarrow \mathbb{Z}_q$,
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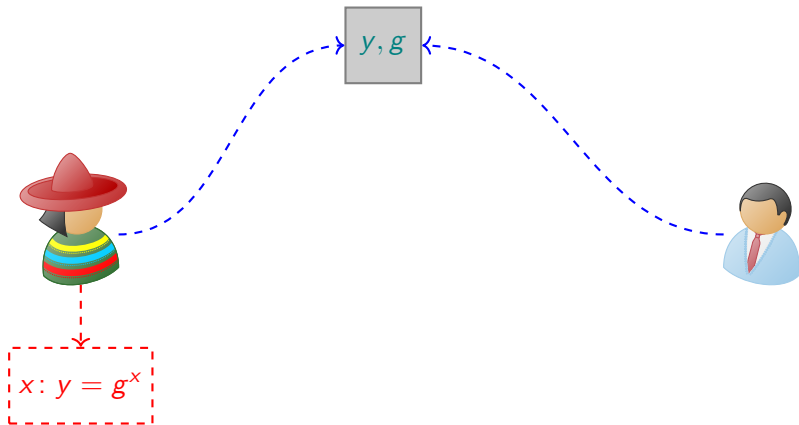
Anamorphic Schnorr

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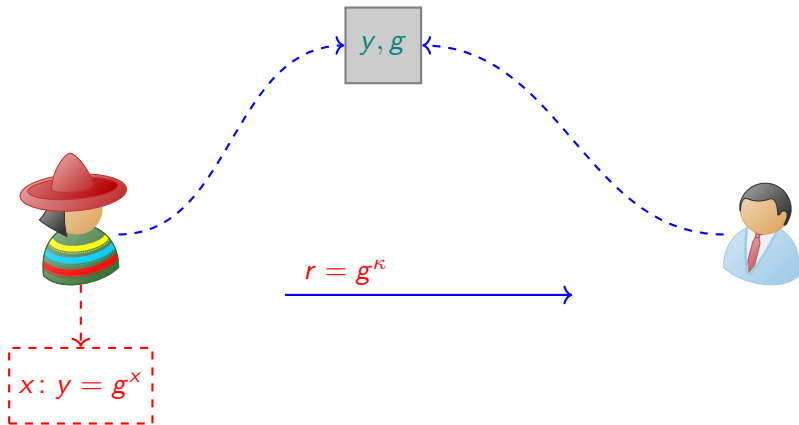
Fishing for randomness

- Set $r = \text{prEnc}(K, \text{amsg})$
 - ▶ need κ to compute s
- Set $\kappa = \text{prEnc}(K, \text{amsg})$
 - ▶ cannot recover κ during verification
 - ★ add x to dkey
 - ★ a many-to-many anamorphic channel

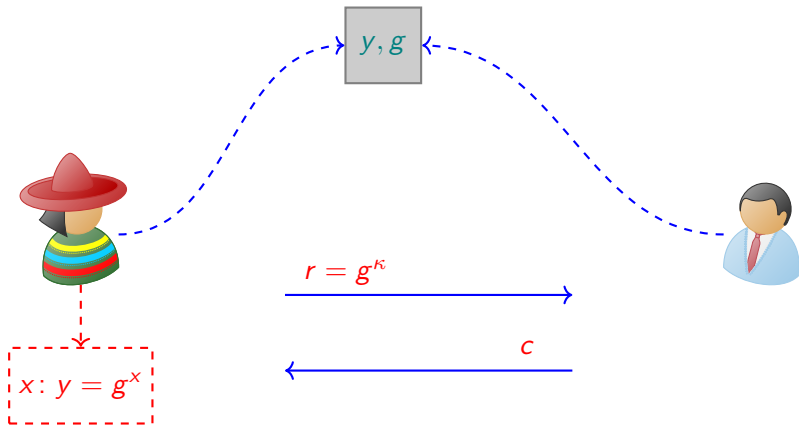
Schnorr's Proof of Knowledge



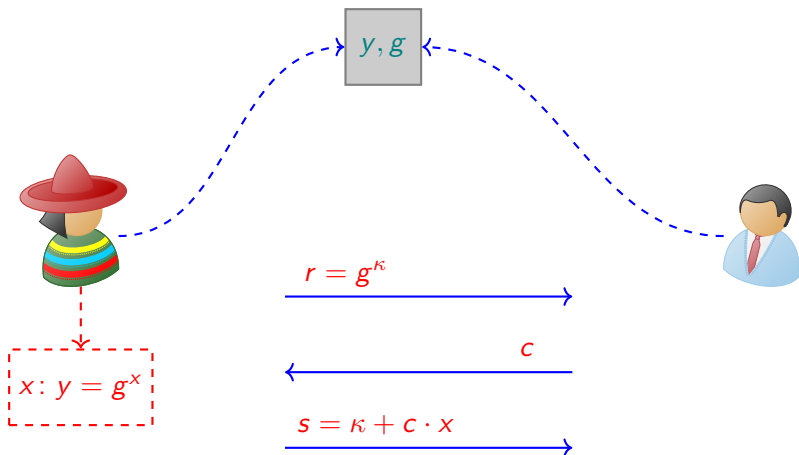
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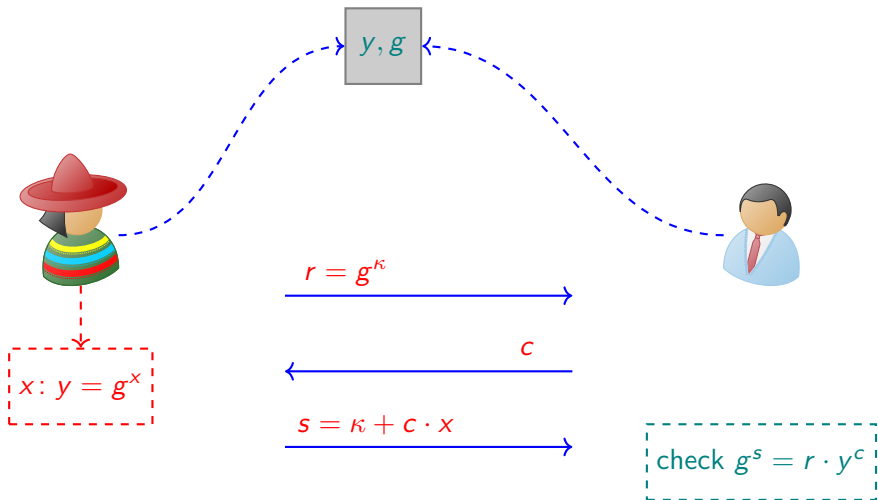
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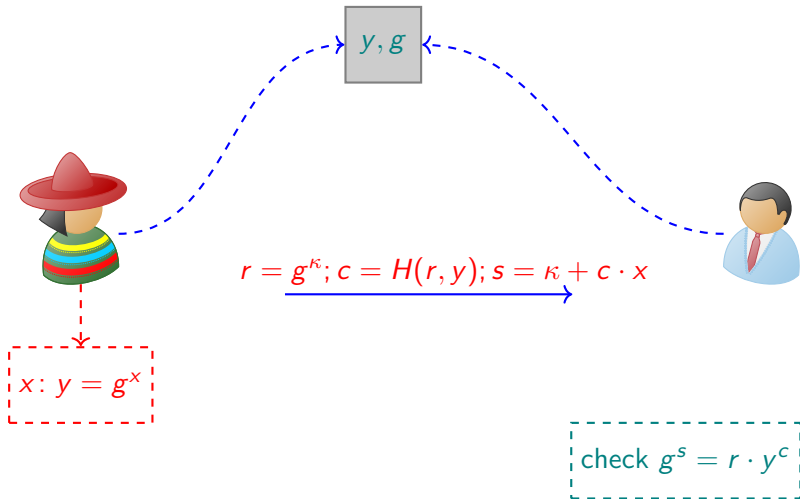
Schnorr's Proof of Knowledge



Schnorr's Proof of Knowledge



Fiat-Shamir



Schnorr's Sig

svk

y, g

msg



$msg, (r, s)$

ssk $x: y = g^x$
 $r = g^{\kappa}$

$$s = \kappa + H(r, msg, svk) \cdot x$$

check $g^s = r \cdot y^c$

$c = H(r, msg, svk)$

Anamorphic Schnorr's Sig

svk

y, g

msg



dkey = (K, x)



msg, (r, s)

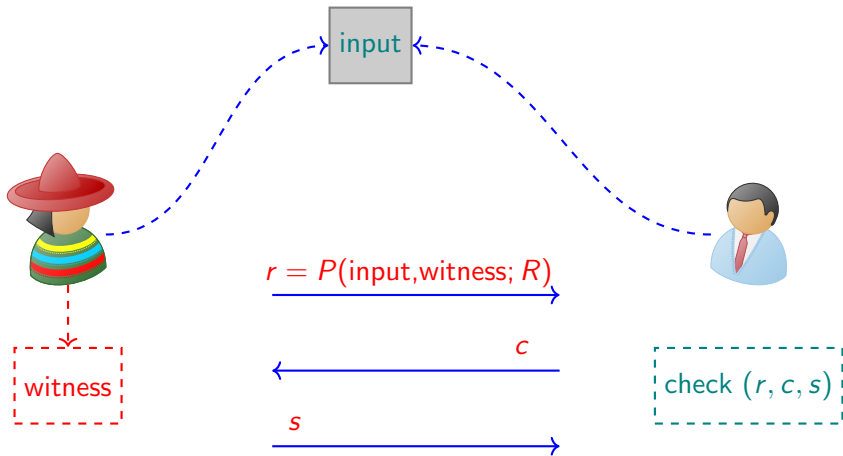
$\kappa = s - H \cdot x$

ssk $x: y = g^x$

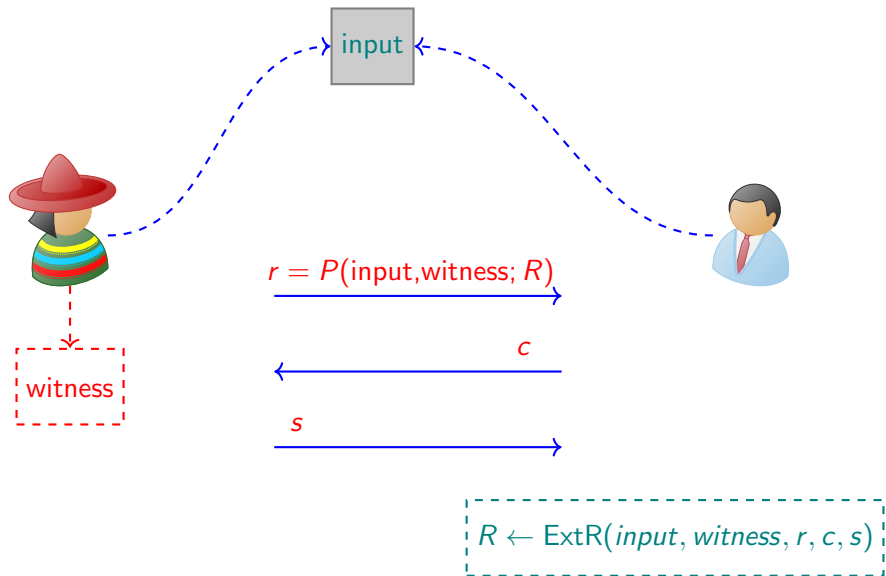
$\kappa = \text{prEnc}(K, \text{amsg})$

$r = g^{\kappa}$

$s = \kappa + H(r, \text{msg}, \text{svk}) \cdot x$



Sufficient condition for Anamorphism



Fiat-Shamir preserves Anamorphism

Fiat-Shamir Heuristics

How to construct a signature scheme from a 3-round interactive proof

Fiat-Shamir preserves Anamorphism

If 3-round interactive proof is **anamorphic** the resulting signature scheme is also **anamorphic**

The Naor-Yung Transformation

Lamport's Tagging Scheme [1979]

- **Key Generation algorithm** $LKG(1^\lambda, 1^\ell)$
 - ▶ for $j = 1, \dots, \ell$.
 - ★ $x_{0,j}, x_{1,j} \leftarrow \{0, 1\}^\lambda$
 - ★ $y_{0,j} = f(x_{0,j})$ and $y_{1,j} = f(x_{1,j})$
$$Lvk = ((y_{0,j}, y_{1,j}))_{j=1}^\ell \text{ and } Lsk = ((x_{0,j}, x_{1,j}))_{j=1}^\ell.$$
- **Signing algorithm** $LSig(m_1, \dots, m_\ell, Lsk)$
 - ▶ $\Sigma = (x_{m_j,j})_{j=1}^\ell$.
- **Verification algorithm** $LVerify(\Sigma, m_1, \dots, m_\ell, Lvk)$
 - ▶ check $f(s_j) = y_{m_j,j}$ for $j = 1, \dots, \ell$.

We have a problem

- **Signing is deterministic**
 - ▶ no randomness to be extracted by the verifier
- There is randomness in the verification key: the $x_{b,j}$'s
- We can embed the **anamorphic message** amsg as $x_{0,1} = \text{prEnc}(K, \text{amsg})$ and $x_{1,1} = \text{prEnc}(K, \text{amsg})$
 - ▶ anamorphic message to be determined at key generation time
 - ▶ **weakly anamorphic**
- It is a one-time signature
- We are going to be fine
 - ▶ for one-time signatures key generation time “*coincides*” with signing time

The Naor-Yung transform

NY Lifting

- one-time to multi-time signature

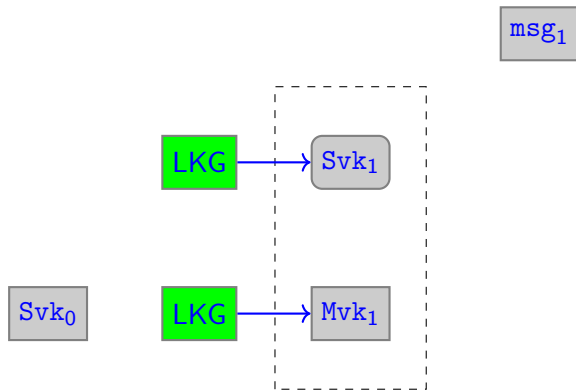
- weakly anamorphic to fully anamorphic

The Naor-Yung transform

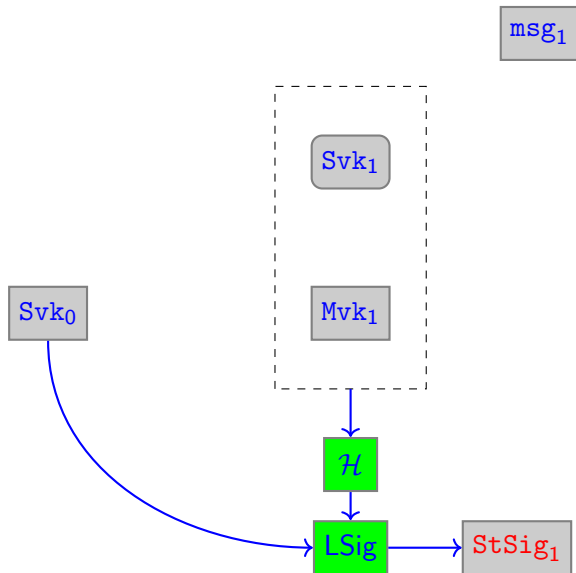
msg_1

Svk_0

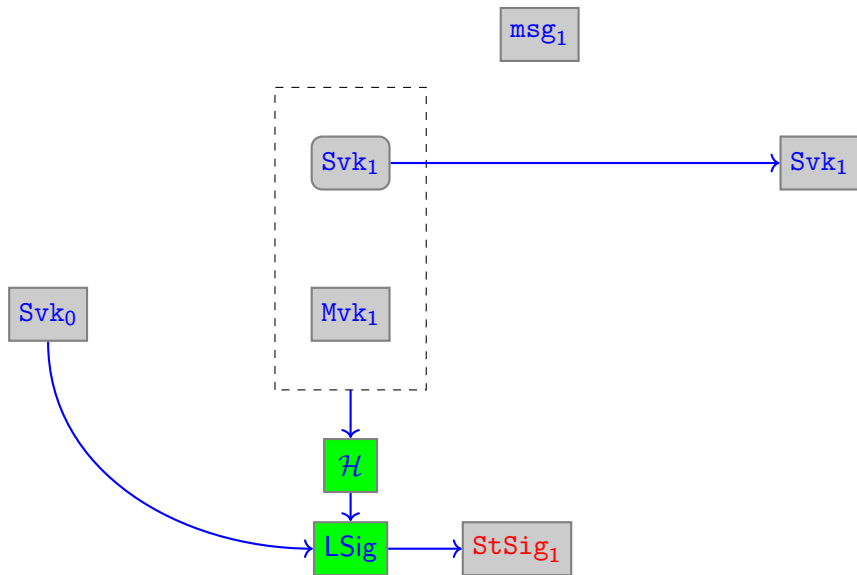
The Naor-Yung transform



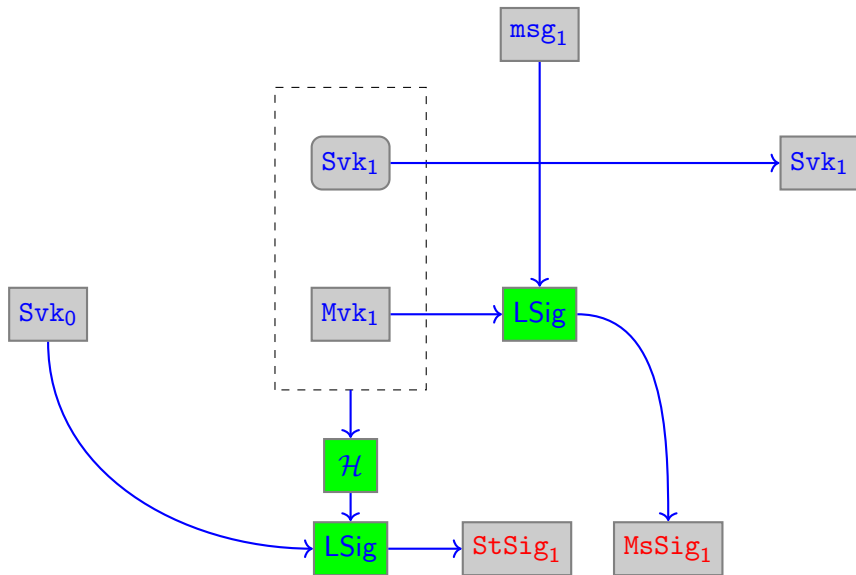
The Naor-Yung transform



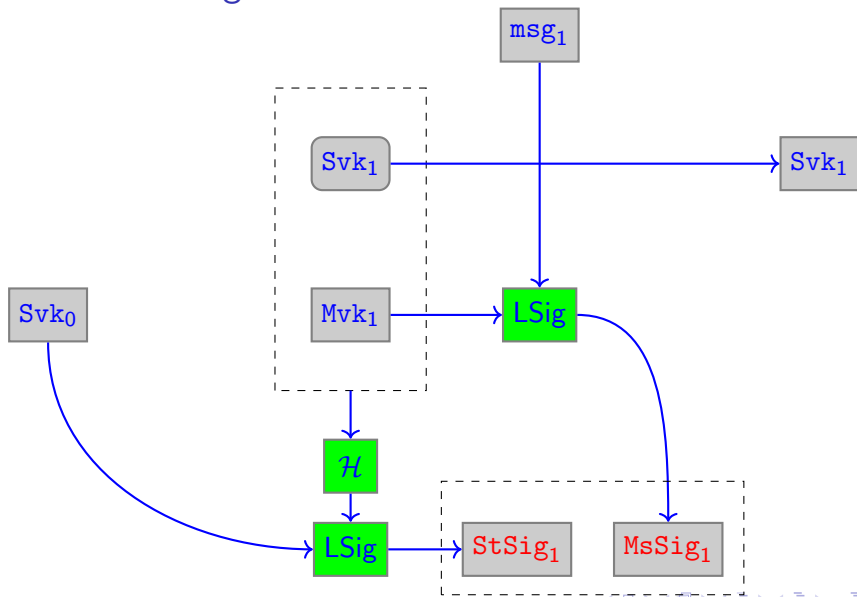
The Naor-Yung transform



The Naor-Yung transform



The Naor-Yung transform



Anamorphic Lifting via the Naor-Yung transform

LKG



Svk_0

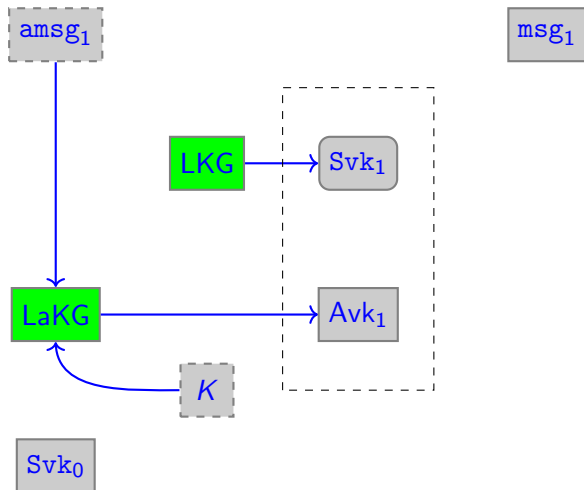
Anamorphic Lifting via the Naor-Yung transform



K

Svk_0

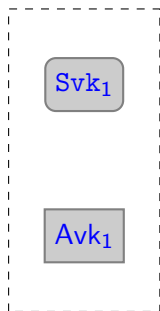
Anamorphic Lifting via the Naor-Yung transform



Anamorphic Lifting via the Naor-Yung transform

$amsg_1$

msg_1



Svk_0

\mathcal{H}

$LSig$

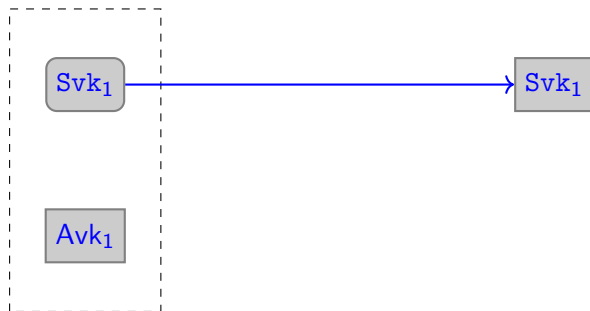
$StSig_1$



Anamorphic Lifting via the Naor-Yung transform

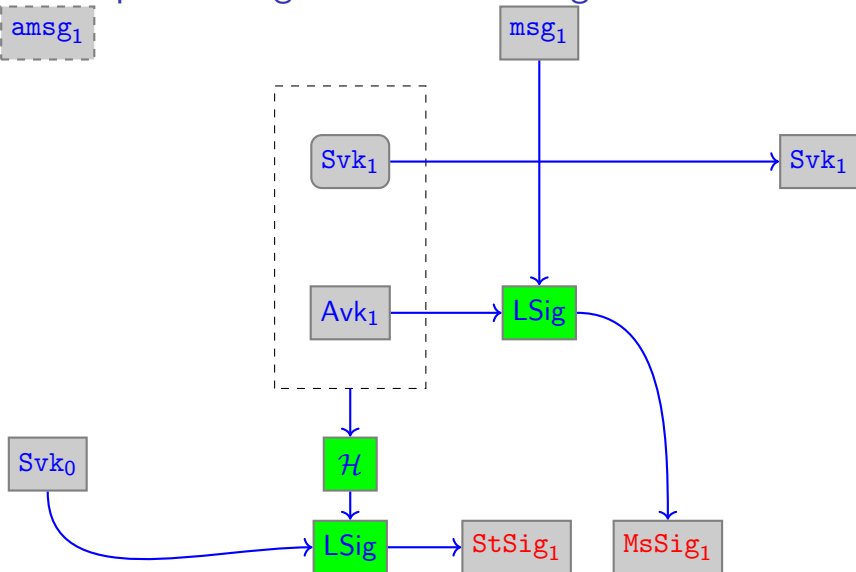
$amsg_1$

msg_1



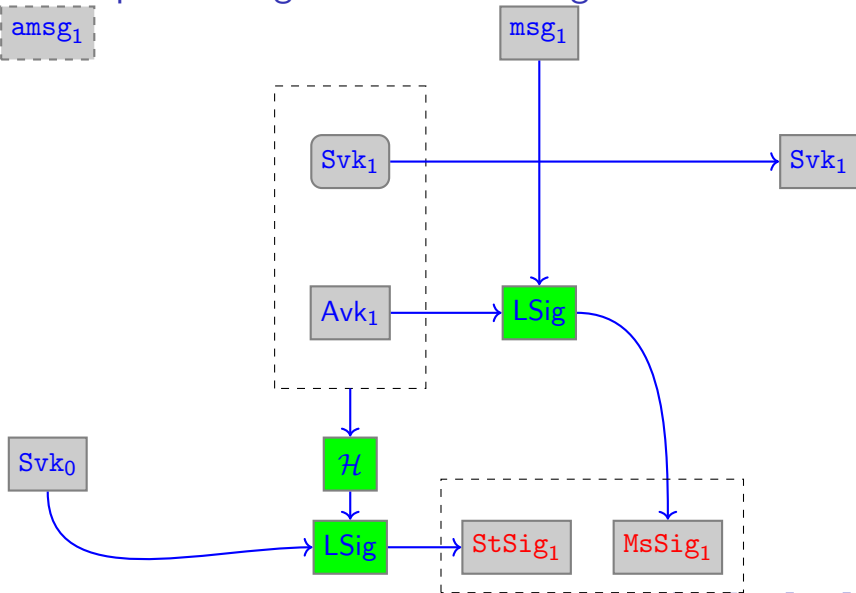
Anamorphic Lifting via the Naor-Yung transform

$amsg_1$



Anamorphic Lifting via the Naor-Yung transform

$amsg_1$



Anamorphism of NY

Lifting

If *universal one-way hash functions* exist, any *weakly anamorphic one-time signature* can be lifted to a *fully anamorphic multi-time signature*.

Naor-Yung-Lamport-Rompel

If *one-functions* exist then there exists a *fully anamorphic multi-time signature scheme*.

One-to-Many Anamorphism of NY

Separability of K

- K and (Svk_0, Ssk_0) are independent
- only need K to extract $amsg$
- K will not help produce signatures

Technical summary

- the new notion of **anamorphic signature**
- **theoretical properties**
- **two flavors:**
 - ▶ *one-to-many anamorphic* communication
 - ★ **dkey** allows decryption but not signature
 - ▶ *many-to-many anamorphic* communication
 - ★ **dkey** allows decryption and signature
- **one general technique**
 - ▶ extract randomness and replace with ciphertext

Technical Summary

- two design paradigms for signatures preserve anamorphism
 - ▶ Fiat-Shamir turns 3-round protocols into signatures in the ROM
 - ★ If prover randomness can be extracted, then resulting signature is anamorphic
 - ★ Schnorr, [Beth 88], [Guillou+Quisquater90], [Ong+Schnorr90], [Brickell+McCurley91], [Girault91], [Okamoto93],[Pointcheval95],[Stern94]
 - ★ Need the witness to extract (i.e., the signing key).
 - ★ Many-to-Many Anamorphism
 - ▶ Naor-Yung turns one-time signatures into many-time signatures in the standard model (assume one-way functions)
 - ★ If one-time signature enjoys a weak form of anamorphism, resulting many-time is fully anamorphic
 - ★ Lamport, BC, HORS
 - ★ One-to-Many Anamorphism
- Applications to schemes using digital signatures
 - ▶ Canetti-Halevi-Katz CCA encryptions scheme uses a signature scheme
 - ▶ the transformation preserves anamorphism

Conclusions

- disallowing encryption is not sufficient
- must disallow message authentication too
 - ▶ complete disruption of communication
 - ▶ not clear who is talking to whom
- or disallow randomized signatures
 - ▶ more to come about this...
- dictator will not care
 - ▶ just give me **dkey** or else...
 - ▶ if no **dkey** then can't surrender it...
- technical evidence that a democracy cannot actually control communication
 - ▶ unless, that is, it ceases to be a democracy

- Giuseppe Persiano, Duong Hieu Phan, Moti Yung: *Anamorphic Encryption: Private Communication against a Dictator*. IACR Cryptol. ePrint Arch. 2022: 639 (2022). Eurocrypt '22
- Mirek Kutylowski, Giuseppe Persiano, Duong Hieu Phan, Moti Yung, Marcin Zawada: *The Self-Anti-Censorship Nature of Encryption: On the Prevalence of Anamorphic Cryptography*. IACR Cryptol. ePrint Arch. 2023: 434 (2023). PETS '23
- Mirek Kutylowski, Giuseppe Persiano, Duong Hieu Phan, Moti Yung, Marcin Zawada: *Anamorphic Signatures: Secrecy From a Dictator Who Only Permits Authentication!* IACR Cryptol. ePrint Arch. 2023: 356 (2023). CRYPTO '23