# Anamorphic Encryption: Current Developments Private Communication against a Dictator

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Results from Eurocrypt 2022 paper, https://ia.cr/2022/639 and work in progress

All joint work with Duong Hieu Phan, Moti Yung

## Privacy as a Human Right

#### UDHR, Article 12: (1948)

No one shall be subjected to arbitrary interference with his privacy, family, home or **correspondence**,...

## End to End Encryption

- Cryptography has been very successful in providing tools for encrypting communication
  - The Signal protocol and app

But its success relies on two assumptions that might be challenged in dictatorial states

## The receiver-privacy assumption

Encryption guarantees message confidentiality only with respect to parties that do not have access to the receiver's private key

The receiver-privacy assumption

The receiver keeps his secret key in a private location

## The sender-freedom assumption

A ciphertext carries the message that was provided as an input, not the one that the sender wishes to encrypt

The sender-freedom assumption

The sender is free to pick the message to be encrypted

## Receiver privacy and Sender freedom

- Both assumptions are realistic for "normal" settings
- No wonder Encryption has been developed under these assumptions
  - with no explicit mention
- In a dictatorship, instead
  - No receiver privacy: citizens might be invited to surrender their private keys



No sender freedom: citizens might be invited to send messages to international newspapers to make the dictator look good

## OK...two more assumptions

Why is this a problem?

#### Theorem

Assume existence of one-way functions and receiver privacy. Then, there exist secure symmetric encryption schemes.

#### Two assumptions

- Existence of one-way functions
- Ability to hide my key

## Law of Nature vs Normative Prescription

• Assumption of the existence of one-way functions comes from *our current scientific understanding of Nature* 

- if true, it is enforced by Nature
- it might be false but then it is false for all

#### • Receiver privacy is a *norm*:

- it is enforced by political power
- it can be changed by law, decree, force
- it could change for some but not for all

## Not only dictators...

#### Various attempts to regulate, limit, cripple encryption

## Crypto Wars

Presently, anyone can obtain encryption devices for voice or data transmissions. [...] if criminals can use advanced encryption technology in their transmissions, electronic surveillance techniques could be rendered useless because of law enforcement's inability to decode the message.

> Howard S. Dakoff *The Clipper Chip Proposal* J. Marshall L. Rev., 29, 1996.

# Crypto Wars

- Combination of cryptographic tools and normative prescription
- From [Micali 1992] to [Green-Kaptchuk-van Laer 2021]
  - Rely on the existence of an independent judiciary system (missing in a Dictatorship!)
- Several related concepts
  - Kleptography [Young-Yung 97]
  - Subvertable encryption
  - Steganography (see later)

# Crypto Wars

Several arguments have been made against restricting encryptions:

- the bad guys can utilize other encryption systems
- all other encryption schemes must be declared illegal
  - what qualifies as an encryption scheme? e.g., chaffing and winnowing
- it creates a natural weak systems security point

All these arguments are indirect and non-technical

We wish to give technical evidence that it is *futile* to try to restrict encryption

Resistence is futile

# Our approach for receiver privacy

#### Constraints

- If the dictator has the secret key sk, it can decrypt and read messages
- But only messages encrypted with respect to sk can be decrypted

#### Our approach

- A ciphertext is associated with two secret keys  $sk_0, sk_1$
- share sk1 with your friend
- A ciphertext carries two plaintexts  $m_0, m_1$ , one for each key
- ...and there is no second key
  - at least, that's what the dictator thinks
  - when dictator asks for keys, give him sk<sub>0</sub> because there is only one key...

## Anamorphic Encryption

- $\mathcal{E} = (KG, Enc, Dec)$  can be used
  - in normal mode.
    - one public key PK, one secret key sk
    - one ciphertext ct, one plaintext m
  - or in *anamorphic* mode: (aKG, aEnc, aDec)
    - one public key PK, two secret keys sk<sub>0</sub>, sk<sub>1</sub>
    - one ciphertext ct, two plaintexts m<sub>0</sub>, m<sub>1</sub>
    - $sk_0$  decrypts ct to  $m_0$  and  $sk_1$  decrypts ct to  $m_1$
- When in anamorphic mode and dictator asks for secret key
  - sk<sub>0</sub> is released
  - dictator has no reason to believe that  $sk_1$  exists
  - dictator can only read m<sub>0</sub>

# Implementing Our Approach

#### Normal mode:

- modify Enc to append a string au of  $\ell$  random bits
- ciphertext  $ct = (ct_0, \tau)$
- one secret key sk output by KG

#### Anamorphic mode:

- generate a sk1 for (KG', Enc', Dec') encryption scheme with pseudo-random ciphertexts
- to encrypt  $m_0$  and  $m_1$ 
  - Encrypt m<sub>0</sub> by running Enc and obtain ct<sub>0</sub>
  - Encrypt  $m_1$  by running Enc' and obtain  $ct_1$
  - Output  $ct = (ct_0, \tau)$  with  $\tau := ct_1$

**Note:** In anamorphic mode there is a secret key generated by KG' shared behind the dictator's back.

## This does not work!!

- We just designed an encryption scheme that is secure without assuming receiver privacy and/or sender freedom
- What is the dictator going to do?
  - It will be considered illegal
  - The simple act of using the new scheme will be self accusatory
  - The encryption scheme and its use will be seen as provocations

Rather, we should look at existing schemes to see if they can be used to defeat the dictator

Existing schemes cannot be disallowed as there are legitimate uses for them. Legitimate, even for the dictator.

## Our thesis

## Our thesis

- Regulating/crippling encryption is technically futile
  - Not because we can construct Anamorphic Encryption
  - But because Anamorphic Encryption is already among us
- The more schemes are found to be anamorphic, the stronger our thesis

# Rejection Sampling Encryption

Hopper, Langford, von Ahn [CRYPTO02] Bellare, Paterson, Rogaway [CRYPTO14]

Normal mode

- $\mathcal{E} = (KG, Enc, Dec)$  any encryption scheme
- Bob has (PK, sk) and makes PK public
- Alice computes ct = Enc(PK, "Glory to our Leader")
- Dictator decrypts ct using sk

#### Anamorphic mode

- Alice and Bob share a randomly chosen seed K for a PRF  $\mathcal{F}$
- Alice wants to send a bit b to Bob
  - samples ct = Enc(PK, "Glory to our Leader")
  - until  $\mathcal{F}(K, \mathtt{ct}) = b$

# Receiver Anamorphic Encryption Schemes: Syntax

- A receiver anamorphic scheme AME consists of schemes:
  - the normal scheme (AME.KG, AME.Enc, AME.Dec);
  - the anamorphic scheme (AME.aKG, AME.aEnc, AME.aDec);

## Bob deploys AME

Normal: use (AME.KG, AME.Enc, AME.Dec) as a regular public-key encryption scheme

#### Anamorphic Deployment of AME for Alice

- Bob runs (aPK, ask, dkey) ← AME.aKG
- aPK is public, ask is given to  $\mathcal{D}$ , and *double key* is dkey shared with Alice.
- Normal users use AME.Enc and aPK to send messages to Bob.
- Alice wants to send confidential message m1
  - Alice sets  $m_0 =$  "Glory to our Leader"
  - Alice computes  $act \leftarrow AME.aEnc(dkey, m_0, m_1)$
  - $\mathcal{D}$  computes  $m_0 \leftarrow \mathsf{AME}.\mathsf{Dec}(\mathtt{act}, \mathtt{ask})$
  - Bob gets  $m_1 \leftarrow \mathsf{AME.aDec}(\mathtt{act}, \mathtt{dkey})$

#### Note: Alice and Bob share dkey

Rejection Sampling as AME  $\mathcal{E} = (KG, Enc, Dec)$  is the Normal scheme The Anamorphic scheme Key Generation:  $aKG(1^{\lambda})$  $(PK, sk) \leftarrow KG(1^{\lambda}) \text{ and } K \leftarrow \{0, 1\}^{\lambda}$ aPK := PK, ask := sk, dkey := (K, PK)Anamorphic Encryption: aEnc(dkey, m, b) sample  $ct \leftarrow Enc(aPK, m)$  until  $\mathcal{F}(K, ct) = b$ Anamorphic Decryption: aDec(ask,dkey,ct) compute m := Dec(ask, ct)compute  $b := \mathcal{F}(K, ct)$ 

## Modes of Operations

	Key Gen.	Encryption	Decryption
Fully Anamorphic	aKG	aEnc	aDec
Anamorphic with Normal Dec	aKG	aEnc	Dec
Anamorphic with Normal Enc	aKG	Enc	Dec
Normal	KG	Enc	Dec

- The *fully anamorphic mode* to communicate privately with Alice.
- The anamorphic mode with normal decryption is used by  $\mathcal{D}$  to decrypt an anamorphic ciphertext sent by Alice.
- The *anamorphic mode with normal encryption* is used by Charlie, unaware that Bob has an anamorphic key, to send a message to Bob.
- The *normal mode* no privacy guarantee against  ${\cal D}$

## Security notion

Normal game and Fully Anamorphic game are indistinguishable to  ${\cal D}$ 

NormalG<sub>AME, $\mathcal{D}$ </sub>( $\lambda$ )

- Set  $(PK, sk) \leftarrow AME.KG(1^{\lambda})$  and send (PK, sk) to  $\mathcal{D}$ .
- For  $i = 1, \ldots, \text{poly}(\lambda)$ :
  - $\mathcal{D}$  issues query  $(m_0^i, m_1^i)$  and receives  $\mathtt{ct} = \mathsf{AME}.\mathsf{Enc}(\mathsf{PK}, m_0^i)$ .
- Return  $\mathcal{D}$ 's output.

## $\mathsf{FullyAG}_{\mathsf{AME},\mathcal{D}}(\lambda)$

- Set (aPK, ask, dkey)  $\leftarrow$  AME.aKG(1<sup> $\lambda$ </sup>) and send (aPK, ask) to  $\mathcal{D}$ .
- For *i* = 1,..., poly(λ):
  - $D \text{ issues query } (m_0^i, m_1^i) \text{ and receives} \\ ct = AME.aEnc(dkey, m_0^i, m_1^i).$
- Return  $\mathcal{D}$ 's output.

# Anamorphic Encryption Schemes

#### Definition

AME = ((KG, Enc, Dec), (aKG, aEnc, aDec)) is Receiver Anamorphic if

- (KG, Enc, Dec) is a secure encryption scheme
- For any PPT  $\mathcal{D}$ ,

 $|\operatorname{Prob}[\operatorname{NormalG}_{\operatorname{AME},\mathcal{D}}(\lambda) = 1] - \operatorname{Prob}[\operatorname{FullyAG}_{\operatorname{AME},\mathcal{D}}(\lambda) = 1]|$ 

is negligible in  $\lambda$ .

(KG, Enc, Dec) is *anamorphic* if there exists (aKG, aEnc, aDec) such that ((KG, Enc, Dec), (aKG, aEnc, aDec))

is anamorphic.

# Steganography

Steganography enables two parties to embed a secret conversation in a *channel conversation.* Hopper, Langford, von Ahn [CRYPTO02]

#### Stego vs Anamorphic

- Steganography works for every distribution over channel conversations
  - Anamorphic Encryption is Steganography for *channel conversation* consisting of ciphertexts of a secure encryption scheme for which the dictator has decryption keys.
- In Anamorphic Encryption the dictator has access to the secret keys corresponding to all public keys
  - The dictator can break the public-key steganography by von Ahn, Hopper [Eurocrypt 04]

# Receiver privacy

## Feasibility result

Rejection sampling encryption gives a one-bit symmetric encryption scheme whose secure does not rely on the receiver-privacy assumption.

#### Rate

- Rejection Sampling can be extended to any length  $\ell$
- Expected encryption time is exponential in  $\ell$
- If you want encryption to be polynomial, each ciphertext carries  $\Theta(\log \lambda)$  hidden bits

# Exploiting randomness

#### The Goldwasser-Micali Encryption

• Key Generation:  $GM.KG(1^{\lambda})$ 

 $N = p \cdot q$ ,

y, a non-square with Jacobi symbol +1 PK = (N, y), sk = (p, q)

- Encryption of  $b \in \{0, 1\}$ : GM.Enc randomly select  $r \leftarrow Z_N^*$  and output  $ct = r^2 \cdot y^b$
- Decryption of ct: GM.Dec if ct is a square, output 0; else output 1

## How to make GM anamorphic

Let  $\mathcal{E} = (KG, Enc, Dec)$  any encryption scheme with pseudorandom ciphertexts.

 $\begin{array}{l} \mathsf{Key \ Generation:} \ \mathsf{a}\mathsf{KG}(1^{\lambda}) \\ (\mathsf{GM}.\mathsf{PK},\mathsf{GM}.\mathsf{sk}) \leftarrow \mathsf{GM}.\mathsf{KG}(1^{\lambda}) \ \mathsf{and} \ \mathsf{sk} \leftarrow \mathsf{KG}(1^{\lambda}). \\ \mathsf{a}\mathsf{PK} := \mathsf{GM}.\mathsf{PK}, \mathsf{ask} := \mathsf{GM}.\mathsf{sk}, \mathsf{dkey} := (\mathsf{sk}) \end{array}$ 

Anamorphic Encryption: aEnc(dkey, b, m)use  $ct \leftarrow Enc(sk, m)$  as randomness r in the GM.Enc algorithm encrypting b.

Anamorphic Decryption: aDec(GM.sk,dkey,ct) recover r from ct using GM.sk and decrypt it using sk

# Why did it work?

#### Randomness Recoverable Encryption

- the decryption key sk gives the plaintext and (part of) the randomness used to produce the ciphertext
- Paillier, OAEP, OAEP+, NTRU, McEliece are randomness recoverable encryption schemes

# The Naor-Yung Encryption Scheme

#### Normal Mode

- Let  $\mathcal{E} = (KG, Enc, Dec)$  any encryption scheme
- Alice runs KG twice, randomly selects  $\Sigma$  and sets  $\mathtt{PK}=(\mathtt{PK}_0,\mathtt{PK}_1,\Sigma)$  and  $\mathtt{sk}=\mathtt{sk}_0$
- If Bob wants to send "Glory to our Leader" to Alice
  - Compute ct<sub>0</sub> = Enc(PK<sub>0</sub>, "Glory to our Leader")
  - Compute ct<sub>1</sub> = Enc(PK<sub>1</sub>, "Glory to our Leader")
  - ▶ Compute NIZK proof  $\Pi$  that  $ct_0$  and  $ct_1$  carry the same plaintext
  - Set  $ct = (ct_0, ct_1, \Pi)$
- To decrypt ct, Alice
  - ► Checks □ is a valid proof
  - If valid decrypts ct<sub>0</sub> using sk

# The Naor-Yung Encryption Scheme

#### Anamorphic Mode

- Alice runs KG twice, runs the **simulator** to get  $(\Sigma, aux)$  and sets  $PK = (PK_0, PK_1, \Sigma)$  and  $sk = (sk_0, sk_1)$
- dkey := aux is shared with Bob
- If Bob wants to send m<sub>0</sub> = "Glory to our Leader" to the dictator and m<sub>1</sub> = "F\*\*\* our Leader" to Alice
  - Compute ct<sub>0</sub> = Enc(PK<sub>0</sub>, "Glory to our Leader")
  - Compute ct<sub>1</sub> = Enc(PK<sub>1</sub>, "F\*\*\* our Leader")
  - Simulate NIZK proof  $\Pi$  that  $ct_0$  and  $ct_1$  carry the same plaintext
  - Set  $ct = (ct_0, ct_1, \Pi)$
- To decrypt ct, Alice uses sk<sub>1</sub> to decrypt ct<sub>1</sub>
- If asked to surrender her secret key, Alice gives  $sk_0$ 
  - The dictator verifies  $\Pi$ , decrypts  $ct_0$  and reads  $m_0 = \text{`Glory to our Leader''}$

# Why does this work?

## Informal

- NIZK implies that the anamorphic and the normal public keys are indistinguishable
- NIZK+IND CPA imply ciphertexts are indistinguishable
- If asked to surrender secret key, Alice gives  $sk_0$ 
  - PK<sub>1</sub> could be generated without the associated secret key (e.g., El Gamal has this property)
- $(PK_0, PK_1, \Sigma, aux)$  is a symmetric encryption key

## Same reasoning applies to [DDN91] and [Sahai99]

# The Koppula-Waters Encryption Scheme CRYPTO '19

- Key Generation: kw.KG $(1^{\lambda})$ 
  - ▶ Generate  $2\lambda$  pairs (PK<sub>bi</sub>, sk<sub>bi</sub>),  $b \in \{0, 1\}, i \in \{1, ..., n\}$
  - Randomly select  $a_1, \ldots, a_n \leftarrow \{0,1\}^{\lambda}$  and  $B \leftarrow \{0,1\}^{\lambda}$
  - ► Set kw.PK =  $(B, (a_i, PK_{0i}, PK_{1i})_{i=1}^{\lambda})$  and kw.sk =  $(sk_{0i})_{i=1}^{\lambda}$ kw.sk =  $(sk_{0i})_{i=1}^{\lambda}$
- Encryption: kw.Enc(kw.PK, m)
  - ▶ randomly select  $K \leftarrow \{0,1\}^{\lambda}$  and  $(sigK, vK) \leftarrow Sign.KG(1^{\lambda})$
  - set  $c = \mathcal{F}(K, 0) \oplus m$
  - ► for  $i = 1, ..., \lambda$ ★  $\tilde{r}_i = \mathcal{F}(K, i)$   $\tilde{r}_i = \mathcal{F}(K, i)$  and  $v_i \leftarrow \{0, 1\}^{\lambda - 1}$ 
    - **\*** if  $K_i = 0$

 $\begin{aligned} & c_{0,i} = \mathsf{Enc}(\mathsf{PK}_{0i}, 1 | v_i; \tilde{r}_i) \ c_{0,i} = \mathsf{Enc}(\mathsf{PK}_{0i}, 1 | v_i; \tilde{r}_i), \ c_{1,i} = \mathsf{Enc}(\mathsf{PK}_{1i}, 0^{\lambda}) \\ & c_{1,i} = \mathsf{Enc}(\mathsf{PK}_{1i}, 0^{\lambda}), \ c_{2,i} = G(v_i) \end{aligned}$ 

★ if  $K_i = 1$   $c_{0,i} = \text{Enc}(PK_{0i}, 0^{\lambda}) c_{0,i} = \text{Enc}(PK_{0i}, 0^{\lambda}), c_{1,i} = \text{Enc}(PK_{1i}, 1|v_i; \tilde{r}_i)$   $c_{1,i} = \text{Enc}(PK_{1i}, 1|v_i; \tilde{r}_i), c_{2,i} = G(v_i) + a_i + B \cdot vK$ Sign  $(c_i, c_i, c_i, c_i)$  writes  $c_i \in K$ 

• Sign  $(c, (c_{0,i}, c_{1,i}, c_{2,i}))$  using sigK

## Obs0: there are $2\lambda$ public keys Obs1: dictator has only $\lambda$ secret keys $sk_{0i}$

# Making KW19 Anamorphic

## Anamorphic key generation

• keep all sk1i

## Anamorphic Encryption

How to encrypt:

- $m_0 =$  "Glory to our Leader"
- $m_1 = "F^{***}$  our Leader"
- Use kw.Enc to encrypt  $m_0$
- 2 Let *i* be such that  $K_i = 0$ 
  - set  $c_{1,i} = \mathsf{Enc}(\mathsf{PK}_{1i}, m_1)$

**Note1:**  $\Theta(\lambda)$  messages can be sent with v.h.p.

## Note2: No shared information!!!

## Receiver-Privacy Assumption

#### • If sender and receiver have a shared secret

- every encryption scheme can be made anamorphic with logarithmic rate
- every Randomness Recoverable Encryption can be made anamorphic with rate depending on the amount of randomess recovered
- the NIZK based CCA secure encryption schemes à la Naor-Yung can be made anamorphic with constant rate
- If sender and receiver have no shared secret
  - the Koppula-Waters encryption scheme can be made anamorphic with rate greater > 1.

## The Sender-Freedom Assumption

#### • The sender is free to choose the message

#### The dictator can force the sender to send a message of his choice

## Sender Anamorphic Encryption

## The story of Oscar and John

• Oscar, an opposition leader, is "asked" by the Leader to send the following message to some media outlet

 $m_0 =$  "I am fine and in good health"

to a forced public key **f**PK

• Oscar wants also to send message

 $m_1 =$  "I am in prison"

to the public key dPK of a journalist John

 Oscar computes special coin tosses R\* such that by setting ct = Enc(fPK, m<sub>0</sub>; R\*) it holds that

 $m_1 = \mathsf{Dec}(\mathsf{dsk}, \mathsf{ct})$ 

#### No prior shared knowledge is needed between Oscar and John

# Sender Anamorphic vs Deniable Encryption

Deniable encryption:

- applies to the same public key
- is not suitable for dictator setting: It was mentioned in [CDNO97] that deniability is impossible where "*Eve* [the adversary] approaches Alice [the sender] before the transmission and requires Alice [the sender] to send specific messages".
- is impossible for a standard encryptions [CDNO97] (This contradicts our objective to use standard encryptions).

Sender Anamorphic Encryption can be used to provide some form of deniability

- ciphertext is now broadcast over a public channel and not sent on a point to point channel
- denying having sent a message *m* to John under the ciphertext ct, by proving that ct corresponds to a message *m*' sent to Carol.

# Sufficient conditions for Sender Anamorphic with no shared key

Any PKE satisfying the 3 following conditions is sender anamorphic.

- Common randomness property. For any c = Enc(PK, m, r) and any PK', there is a m' such that c = Enc(PK', m', r)
- Message recovery from randomness.

Given the ciphertext and the used randomness, one can recover the corresponding message.

**3** Equal Distribution of Plaintexts.

Given any c in the ciphertext space, for a randomly generated secret key sk: Pr[Dec(sk, c) = 0] = Pr[Dec(sk, c) = 1]

Consequently:

- LWE encryption by Regev, 2005
- Dual LWE encryption by Gentry, Peikert, and Vaikuntanathan, 2008

are sender anamorphic encryption schemes.

## Conclusions

• We introduced two new concepts:

- receiver anamorphic encryption the receiver of a communication is under the dictator's control
- sender anamorphic encryption
   the sender of a message is under the dictator's control
- Anamorphic encryption is not an isolated phenomenon.
- Our results gives technical evidence of the futility of the Crypto Wars
  - the dictator doomed to read Crypto papers and outlaw schemes as they are shown to be *anamorphic*
- How this is going to affect policy, law and other societal aspects is beyond the scope of this work

Thank You