The Curse of the Length
The Case of Encrypted Multi-Maps

Giuseppe Persiano

Università di Salerno

August, 2020

Mitigating Leakage in Secure Cloud-Hosted Data Structures: Volume-Hiding for Multi-Maps via Hashing
by Sarvar Patel, GP, Kevin Yeo, and Moti Yung
CCS '19

Start from the beginning

Reprinted from JOURNAL OF COMPUTER AND SYSTEM SCIENCES

Vol. 28, No. 2, April 1984
Printed in Belgium

Probabilistic Encryption*

SHAFI GOLDWASSER AND SILVIO MICALI

Laboratory of Computer Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

Received February 3, 1983; revised November 8, 1983

Definition of Secure Encryption

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Probabilistic Encryption*

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A new probabilistic model of data encryption is introduced. For this model, under suitable complexity assumptions, it is proved that extracting *any information* about the cleartext from the cyphertext is hard on the average for an adversary with polynomially bounded computational resources. The proof holds for any message space with any probability distribution. The first implementation of this model is presented. The security of this implementation is proved under the intractability assumption of deciding Quadratic Residuosity modulo composite numbers whose factorization is unknown.

1. Introduction

This paper proposes an encryption scheme that possesses the following property:

Whatever is efficiently computable about the cleartext given the cyphertext, is also efficiently computable without the cyphertext.

The fine print

Formal Setting

Let Π be a PKC. Let MG be a message generator. We write M_k for MG$[k]$. Without loss of generality, we assume that all $m \in M_k$ have the same length $l_k = Q(k)$ for some polynomial Q.

The fine print

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Indeed WLOG:

The fine print

Formal Setting

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Indeed WLOG:

Just pad each message in the message space to the maximum length

The fine print

Formal Setting

Let Π be a PKC. Let MG be a message generator. We write M_k for MG$[k]$. Without loss of generality, we assume that all $m \in M_k$ have the same length $l_k = Q(k)$ for some polynomial Q.

Encryption necessarily leaks an upper bound on the length of the plaintext

Incompressibility

Fact of life

Encryption necessarily leaks an upper bound on the length of the plaintext

A direct consequence of Shannon/Kolmogoroff

Structured Encryption

Chase-Kamara 2010

- Data is often organized in Data Structures

Structured Encryption

Chase-Kamara 2010

- Data is often organized in Data Structures
- For efficient retrieval

Structured Encryption

Chase-Kamara 2010

- Data is often organized in Data Structures
- For efficient retrieval
- Storage is outsourced to untrusted server

Structured Encryption   Chase-Kamara 2010

- Data is often organized in Data Structures
- For efficient retrieval
- Storage is outsourced to untrusted server
  - honest but very curious

(Plaintext) Multi-Maps

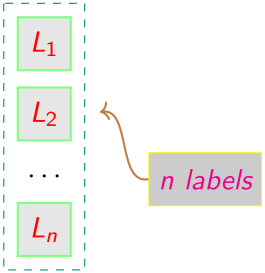
L_1

L_2

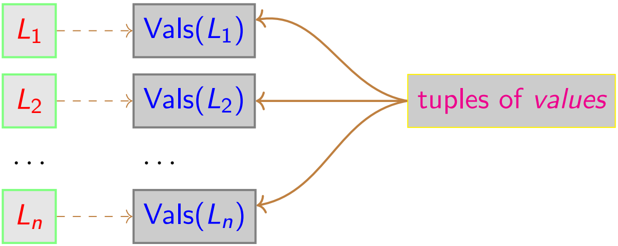
$\cdots$

L_n

(Plaintext) Multi-Maps



(Plaintext) Multi-Maps



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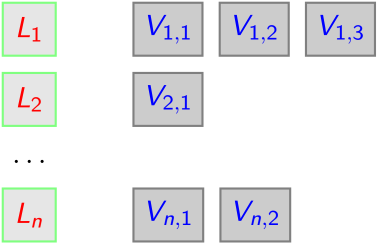

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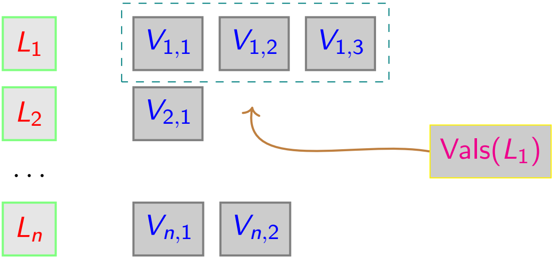

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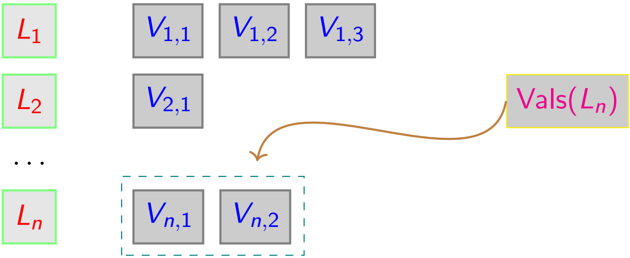

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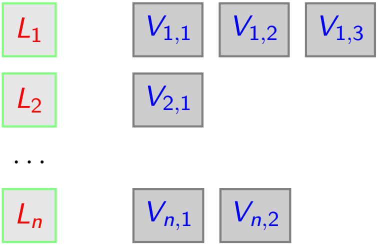
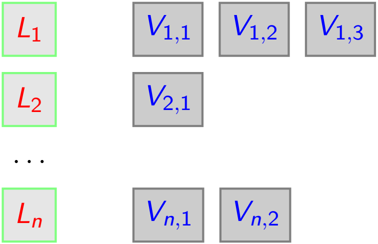


Supported operations

$\mathsf{Init}((L_i, \mathsf{Vals}(L_i))_i)$

$\mathsf{Get}(L) \rightarrow \mathsf{Vals}(L)$

(Plaintext) Multi-Maps



Supported operations

$\mathsf{Init}((L_i, \mathsf{Vals}(L_i))_i)$

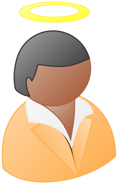
$\mathsf{Get}(L) \to \mathsf{Vals}(L)$

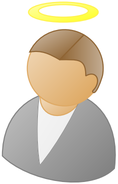
Inverted index

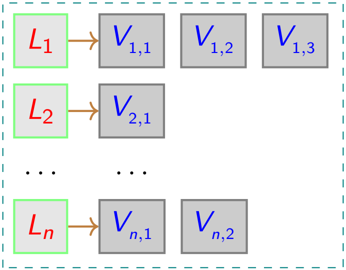
Labels are keywords

Values are documents

Plaintext Multi-Maps



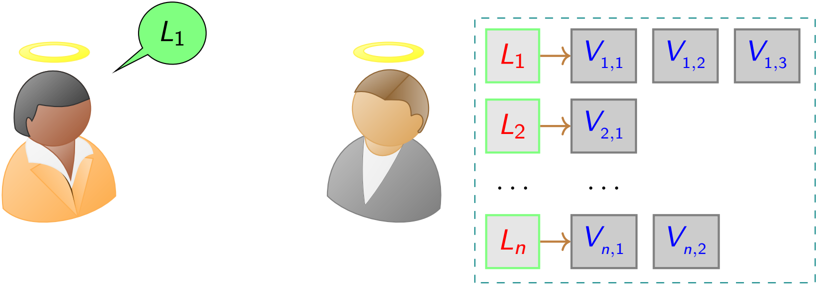




User

Cloud Manager

Plaintext Multi-Maps



User

Cloud Manager

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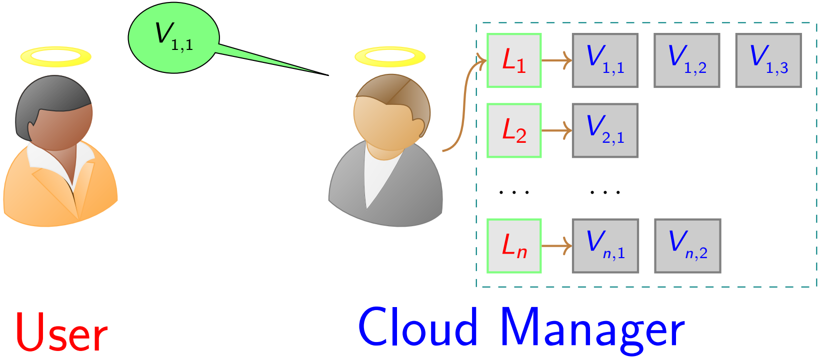
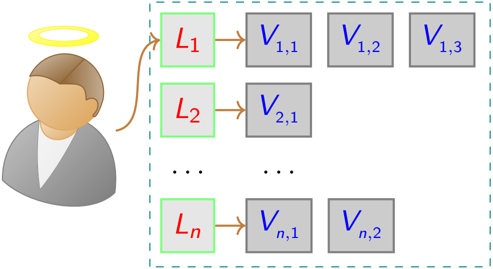


Plaintext Multi-Maps

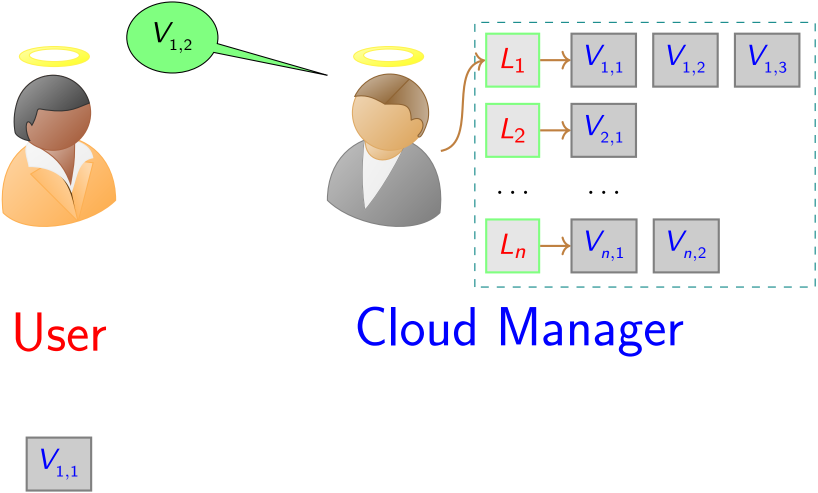


User

Cloud Manager

$V_{1,1}$

Plaintext Multi-Maps



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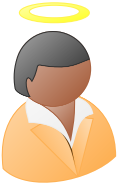

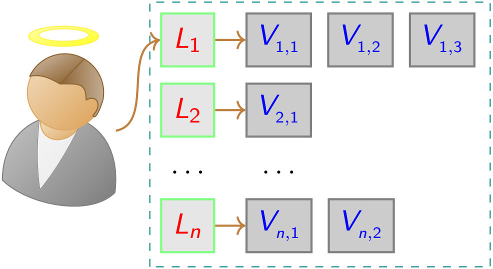

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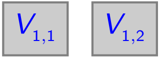
Cloud Manager



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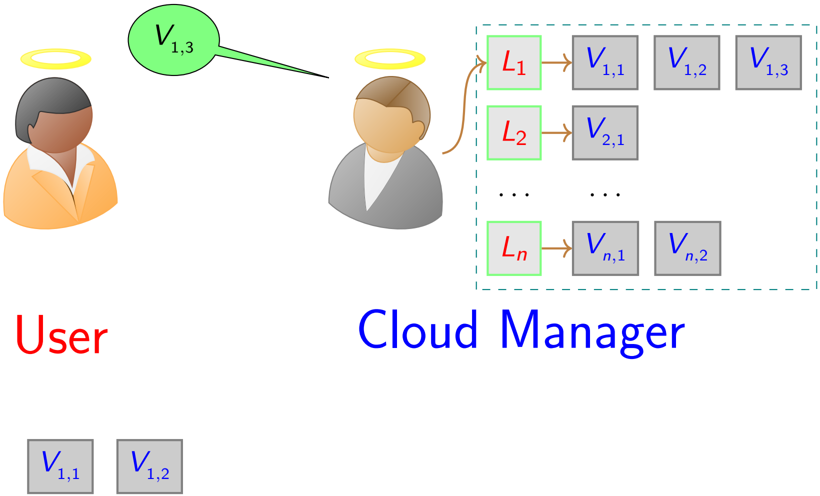

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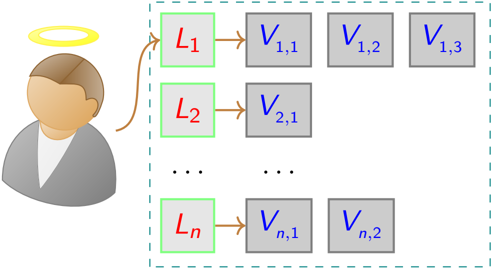
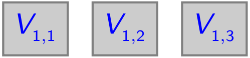
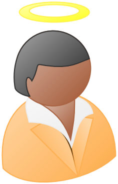


User

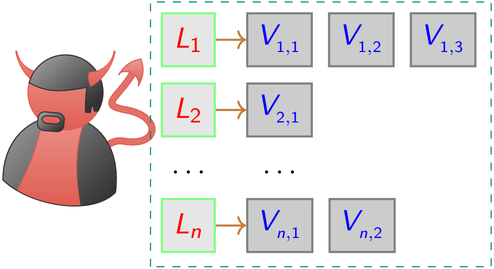
Cloud Manager



Plaintext Multi-Maps with Evil Cloud Manager

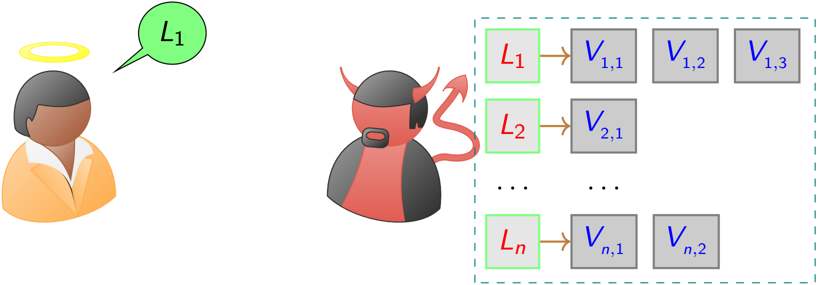


User



Cloud Manager

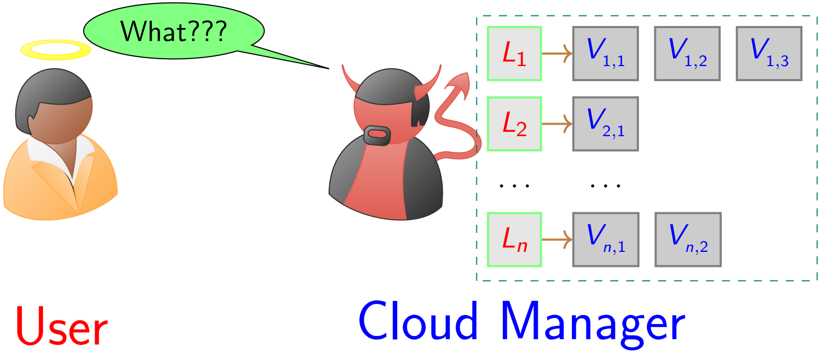
Plaintext Multi-Maps with Evil Cloud Manager



User

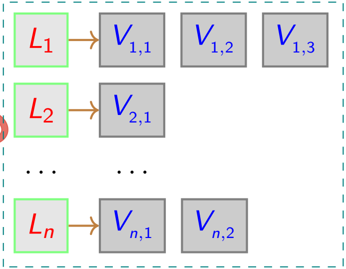
Cloud Manager

Plaintext Multi-Maps with Evil Cloud Manager



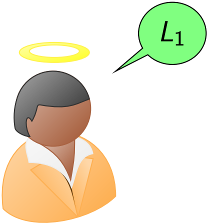
Plaintext Multi-Maps with Honest-but-Curious Cloud Manager

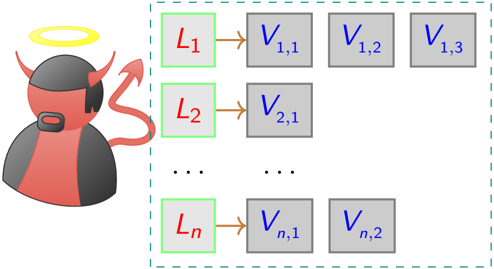
User



Cloud Manager

Plaintext Multi-Maps with Honest-but-Curious Cloud Manager





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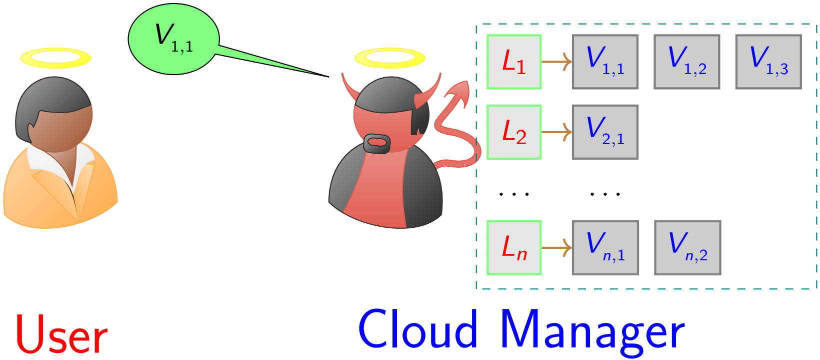

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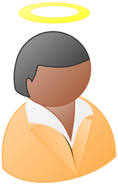
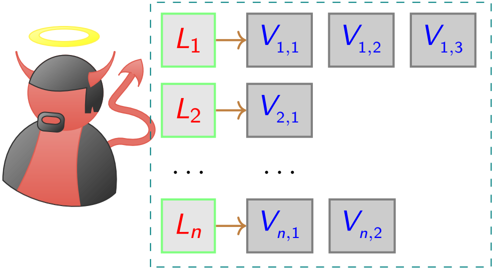


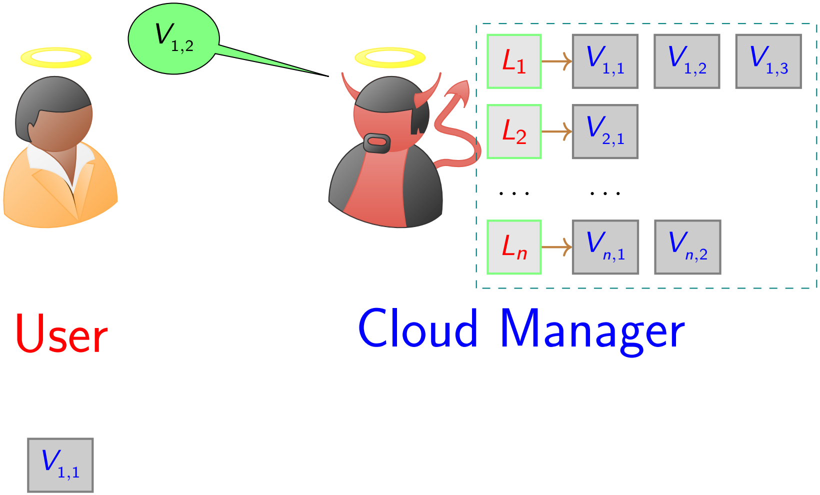


User

Cloud Manager

$V_{1,1}$

Plaintext Multi-Maps with Honest-but-Curious Cloud Manager



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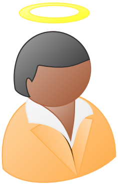
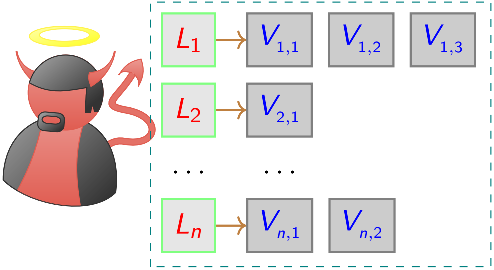
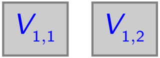




User

Cloud Manager



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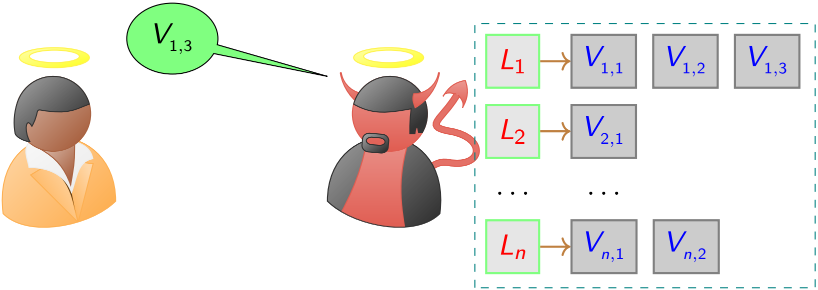
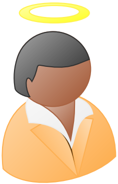


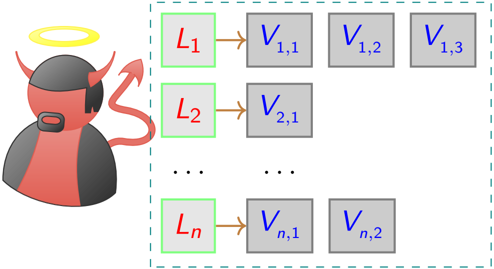
User

Cloud Manager

$V_{1,1}$ $V_{1,2}$

Plaintext Multi-Maps with Honest-but-Curious Cloud Manager





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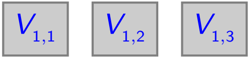

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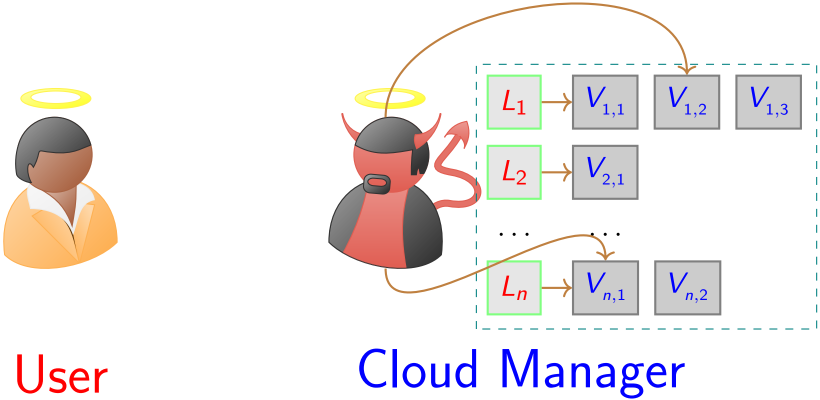

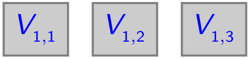

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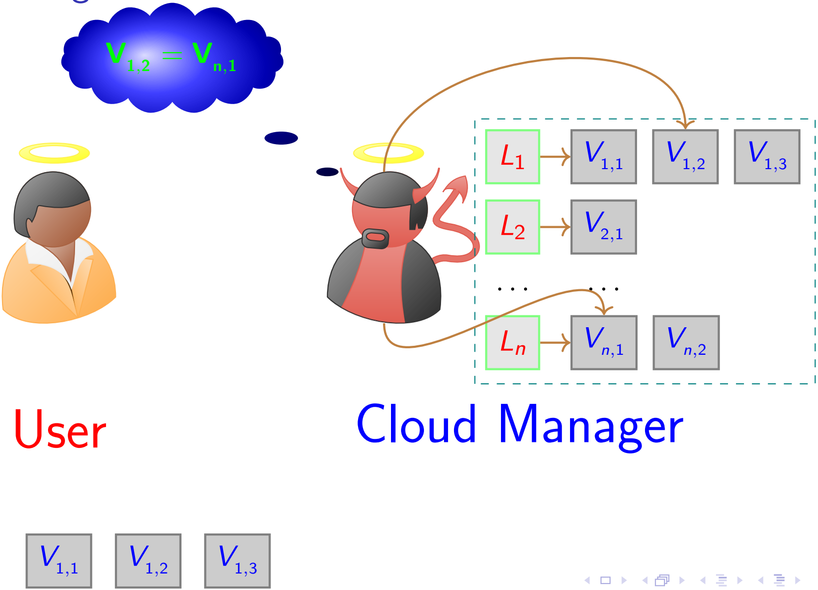
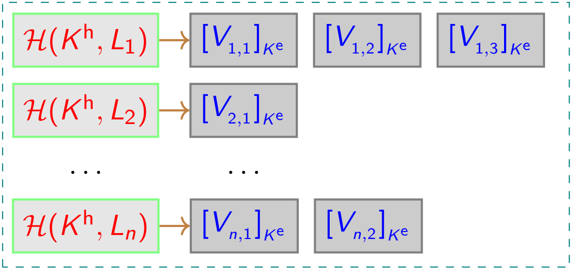


The "Hash-and-Encrypt Approach"



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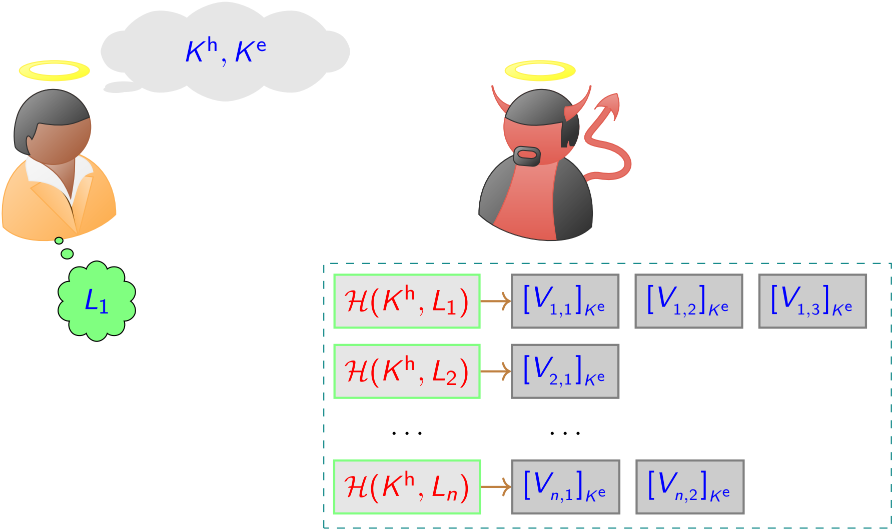

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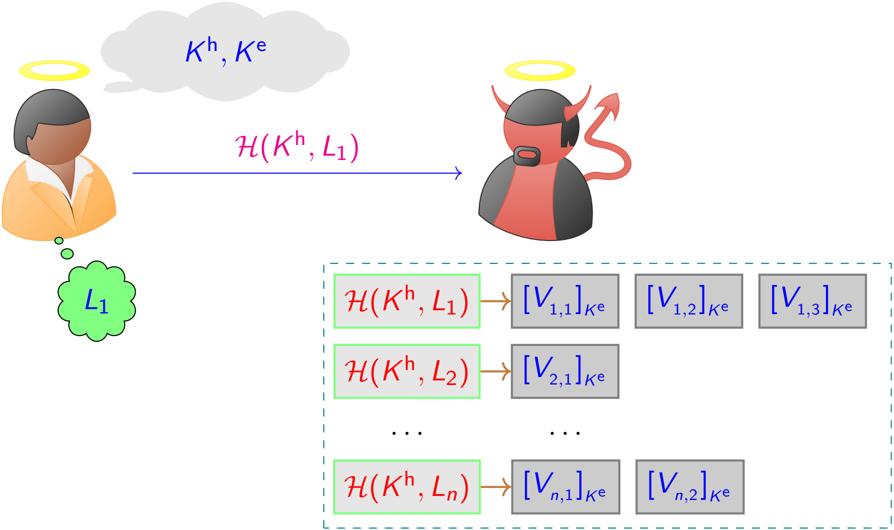

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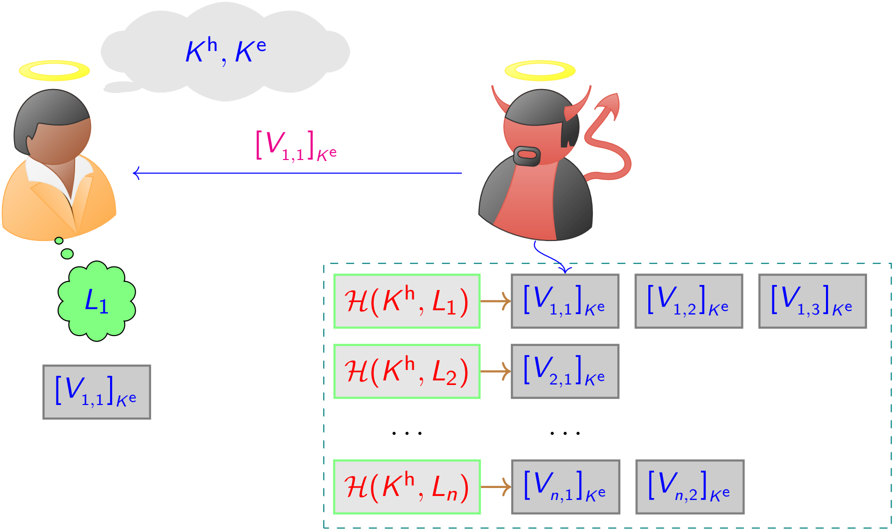

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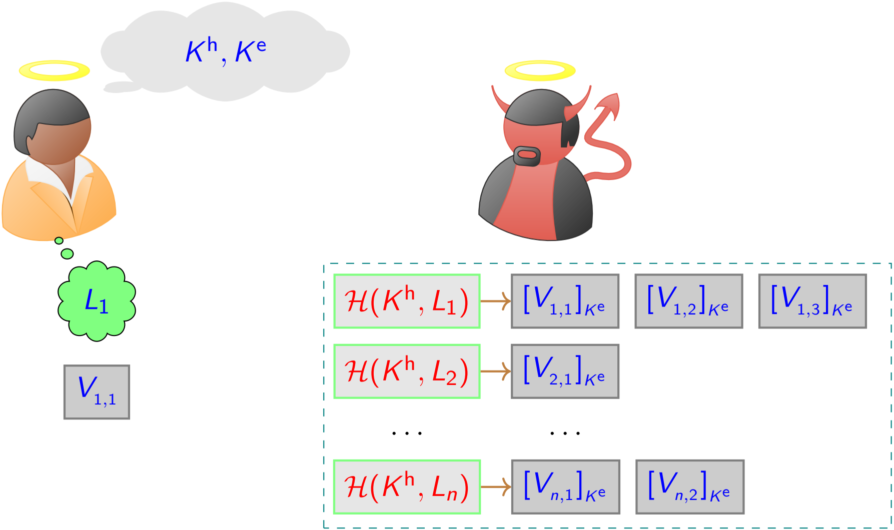

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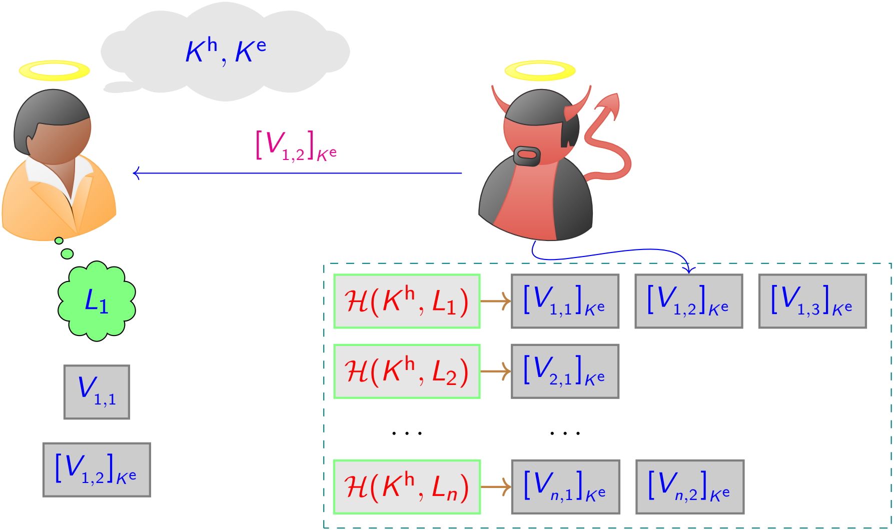

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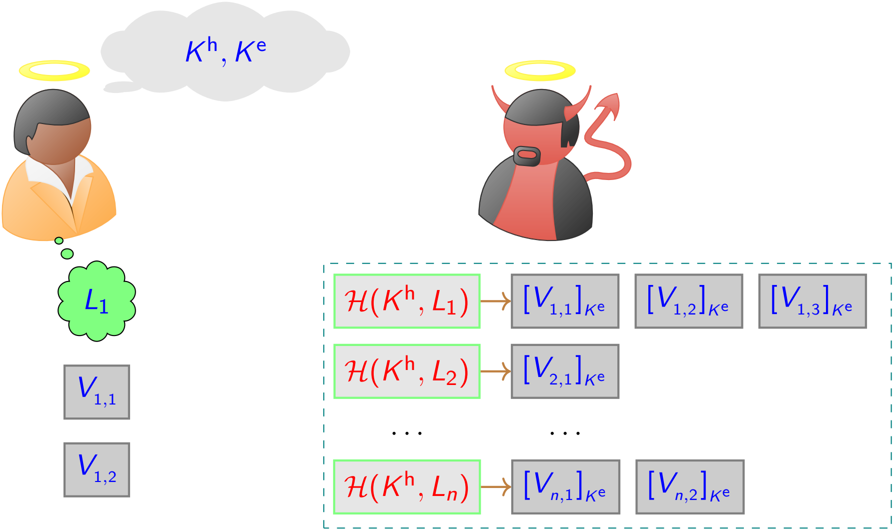

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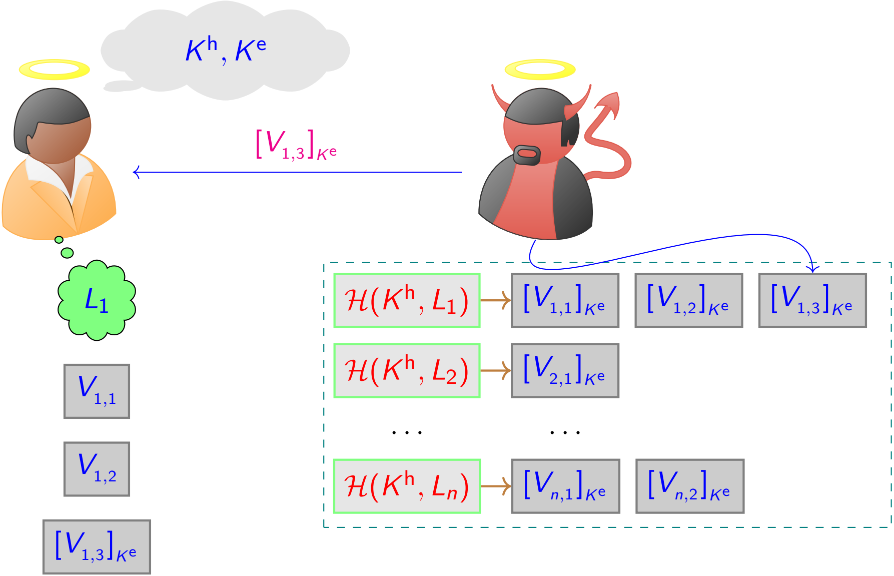

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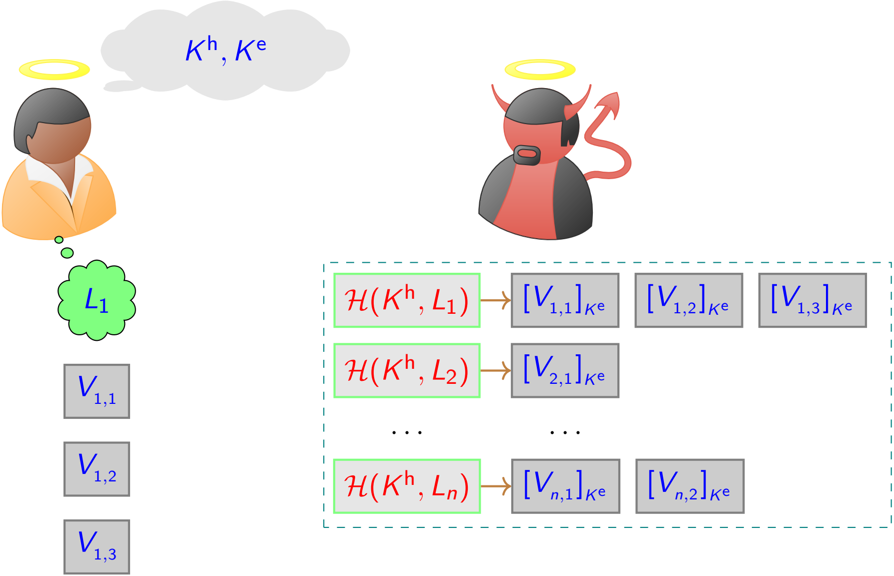
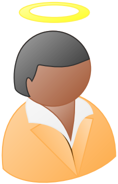
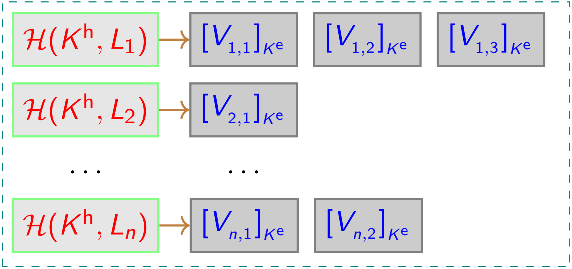


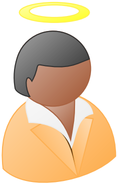
The Leakage: what the Cloud Manager learns





The Leakage: what the Cloud Manager learns

N, number of ciphertexts



$\mathcal{H}(K^{\mathrm{h}}, L_1) \rightarrow [V_{1,1}]_{K^{\mathrm{e}}}$ $[V_{1,2}]_{K^{\mathrm{e}}}$ $[V_{1,3}]_{K^{\mathrm{e}}}$

$\mathcal{H}(K^{\mathrm{h}}, L_2) \rightarrow [V_{2,1}]_{K^{\mathrm{e}}}$

$\dots$ $\dots$

$\mathcal{H}(K^{\mathrm{h}}, L_n) \rightarrow [V_{n,1}]_{K^{\mathrm{e}}}$ $[V_{n,2}]_{K^{\mathrm{e}}}$

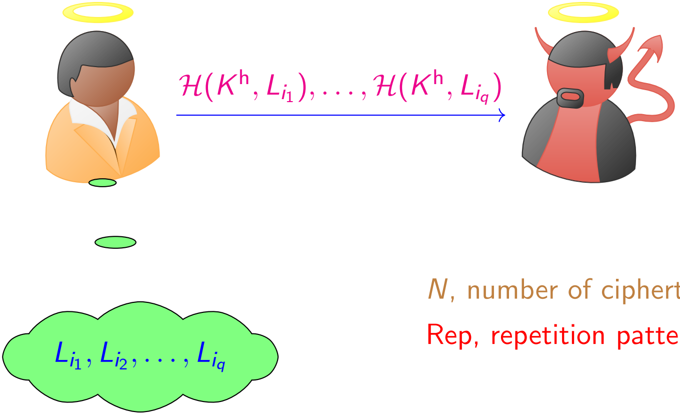
The Leakage: what the Cloud Manager learns

$\mathcal{H}(K^{\mathsf{h}}, L_{i_1}), \ldots, \mathcal{H}(K^{\mathsf{h}}, L_{i_q})$ →

$L_{i_1}, L_{i_2}, \ldots, L_{i_q}$

N, number of ciphertexts

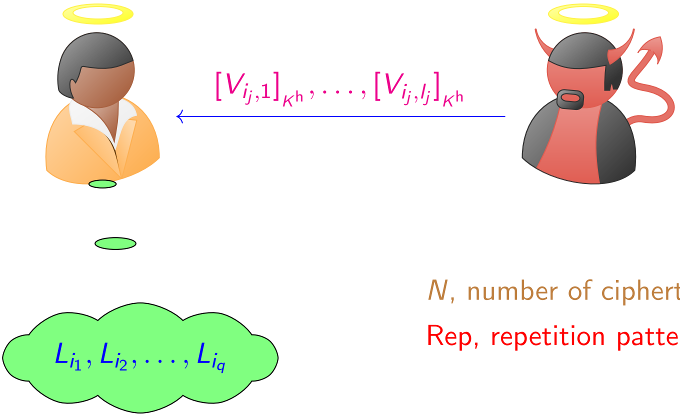
The Leakage: what the Cloud Manager learns



N, number of ciphertexts

Rep, repetition pattern

The Leakage: what the Cloud Manager learns



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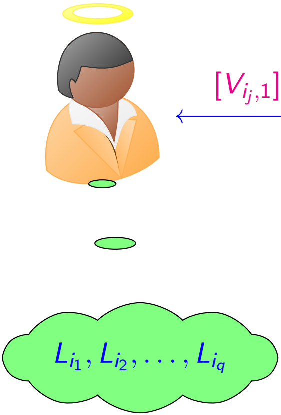
The Leakage: what the Cloud Manager learns

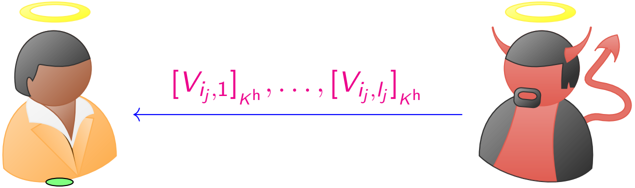


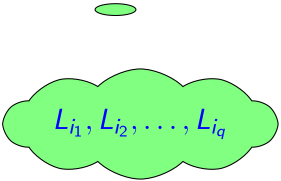
$[V_{i_j,1}]_{K^{\mathsf{h}}}, \ldots, [V_{i_j,l_j}]_{K^{\mathsf{h}}}$

N, number of ciphertexts

Rep, repetition pattern

The Leakage: what the Cloud Manager learns





N, number of ciphertexts

Rep, repetition pattern

l_j, volume of the j-th reply

The Leakage: what the Cloud Manager learns

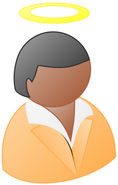
$L_{i_1}, L_{i_2}, \ldots, L_{i_q}$

N, number of ciphertexts

Rep, repetition pattern

l_j, volume of the j-th reply

Padding to maximum length



$\mathcal{H}(K^{\mathsf{h}}, L_1)$ →	$[V_{1,1}]_{K^{\mathsf{e}}}$	$[V_{1,2}]_{K^{\mathsf{e}}}$	$[V_{1,3}]_{K^{\mathsf{e}}}$
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...	...		
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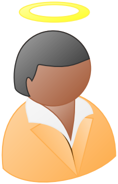

$\mathcal{H}(K^{h}, L_1) \rightarrow [V_{1,1}]_{K^{e}}$ $[V_{1,2}]_{K^{e}}$ $[V_{1,3}]_{K^{e}}$

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$\dots$ $\dots$

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Leakage

n

$\ell_{\max}$

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The Simulator

$$\mathcal{I} = (\mathcal{Q}, \text{Data})$$

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Simulated View

$R_i, ([0]_{K^{\text{e}}}, \ldots, [0]_{K^{\text{e}}})$

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Indistinguishable

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It seems we are done

Implementation of Encrypted Multi-Map

1. That leaks
   - size of the multi-map
   - query repetition pattern
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Densest Subgraph Transform

[Kamara-Moataz '19]

DST

1. We have n bins
2. For each key L_i assign the $\ell_{\max}$ elements to (pseudo)-randomly chosen bins
3. Pad all bins to maximum size $\Theta(\log n)$
4. To retrieve the values for label L_i retrieve the L bins

Query time: $\Theta(L \cdot \log n)$

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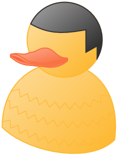
- Security based on hardness of planted densest subgraph

Still unhappy...

Desiderata

1. $\Theta(n)$ server storage in the worst case
2. $\Theta(\ell_{max})$ communication in the worst case
3. Standard complexity assumptions



Blueprint for Volume Hiding Multi-Maps

Dream Data Structure

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- given N items, almost all can be stored using $\Theta(N)$ memory

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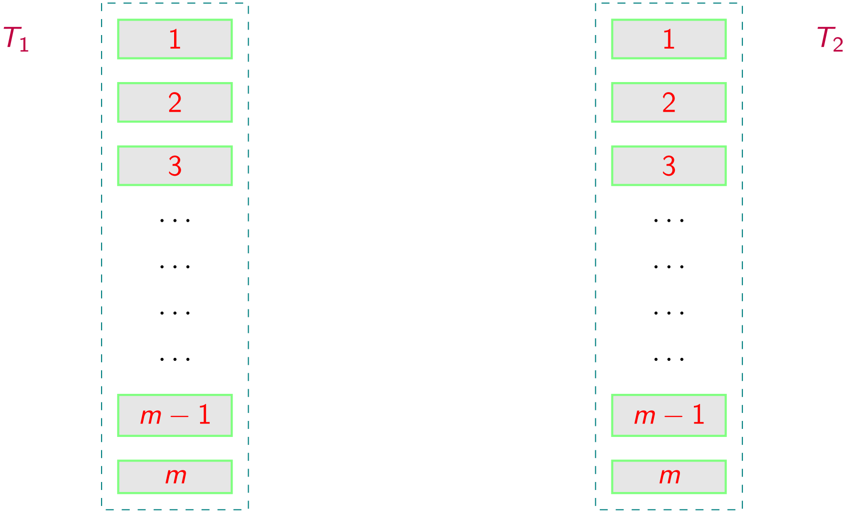
Server memory: $\Theta(N)$, $N = \ell_1 + \ldots, \ell_n$
Query bandwidth: $\Theta(\ell_{\max})$
Client memory: few values

Enter Cuckoo Hashing

The Cuckoo Graph for n items



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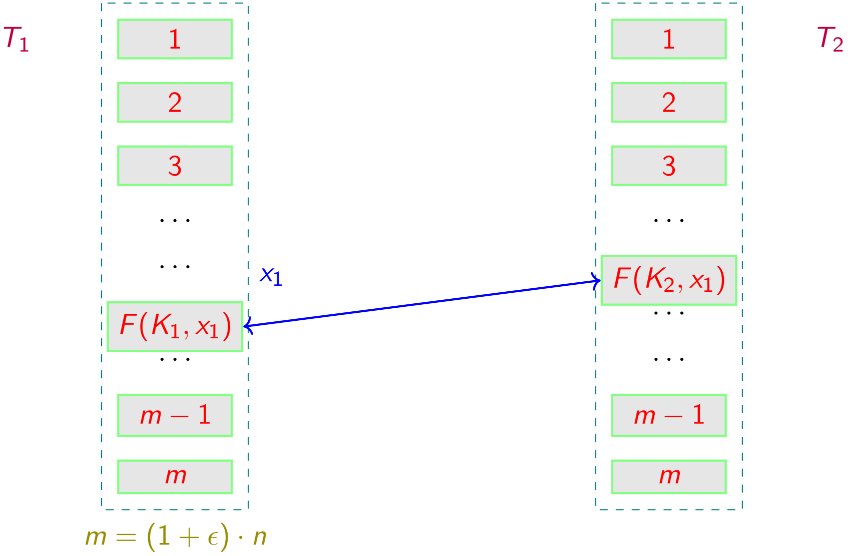

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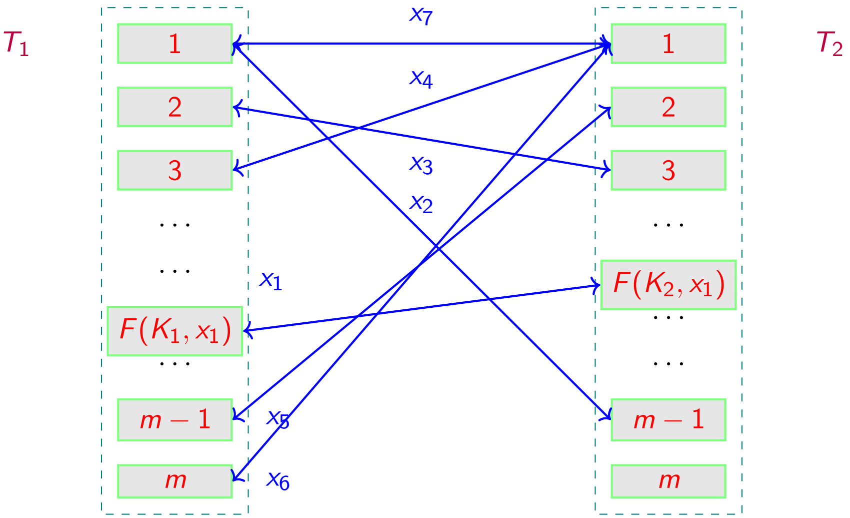

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Theorem (Kirsch-Mitzenmacher-Wieder '09)

The probability that s items are stored in the stash is $O(n^{-s})$.

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- If $M = \Omega(\log n)$ then resulting stash is very small and average insertion time stays constant.

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Encrypted Multi-Maps using Cuckoo Hashing

Algorithm **Init**

Data $= ((L_1, \mathsf{Vals}(L_1)), \ldots, (L_n, \mathsf{Vals}(L_n)))$

1. randomly select seeds K_1, K_2 for PRF F
2. randomly select encryption key K^{e}
3. for $i = 1, \ldots, n$
   - randomly select permutation Π_i over $[1, \ldots, \ell_{\max}]$
   - for $j = 1$ to l_i
     - Add edge labeled $[L_i, V_{i,j}]_{K^{\mathsf{e}}}$ between vertices $F(K_1, (L, \Pi_i(j)))$ and $F(K_2, (L, \Pi_i(j)))$
4. Construct T_1 and T_2 (stored remotely) and stash (stored locally)

Encrypted Multi-Maps using Cuckoo Hashing

Algorithm **Get**

Retrieve values for label L

- for $j = 1, \ldots, \ell_{\mathsf{max}}$:
  - ask for slot $F(K_1, (L, j))$ in table T_1 and slot $F(K_2, (L, j))$ in table T_2
- decrypt all ciphertexts received
- keep the ones $(L, \star)$
- look for the missing ones in the stash

Encrypted Multi-Maps using Cuckoo Hashing

1. Leakage
   - N, total number of values
   - L, maximum volume
   - Rep, query repetition pattern
2. Storage
   - Server: $(2 + \epsilon)N$
   - Client: practically constant
3. Communication
   - Client to Server $2L$ indices
   - Server to Client $2L$ ciphertexts

Security assuming One-Way Functions

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   - Client to Server 2 seeds (by using delegatable PRFs)
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- This very wasteful when there is a huge variation in the length of tuples (e.g., Zipf's law)
- **Question**:
  Can we obtain some meaningful privacy notion that allows for downloading $\leq \ell_{max}$ entries?

(ϵ, δ)-Differentially Private Volume-Hiding Encrypted Multi-Maps

Data^1 and Data^2 differ in the volume of one label L_i

$$|l_i^1 - l_i^2| = 1$$

then

$$\mathrm{Prob}[\mathsf{View}(\mathsf{Data}^1) \in S] \leq e^{\epsilon} \cdot \mathrm{Prob}[\mathsf{View}(\mathsf{Data}^2) \in S] + \delta$$

for all subsets S of views

Cuckoo hashing with DP

To retrieve tuple for label L,

$$\begin{array}{ll} F(K_1, L||1) & F(K_2, L||1) \\ F(K_1, L||2) & F(K_2, L||2) \\ F(K_1, L||3) & F(K_2, L||3) \\ \dots & \dots \\ F(K_1, L||\ell_{max}) & F(K_2, L||\ell_{max}) \end{array}$$

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F(K_1, L||\ell_{\max} + Z_L) & F(K_2, L||\ell_{\max} + Z_L)
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Z_L is the *noise* from Laplacian$(O(1/\epsilon))$ dist.

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Make sure $T \geq |Z_L|$

$|Z_L| > \log n$ with negligible probability

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Data is Sanitized

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- Sampled once and stored over the server
- We need a dictionary to store it
- No volume leakage

Experiments

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Volume Hiding with dPRF vs DST

Cuckoo Hash with $m = 1.3n$

Give up insertion after $5 \log n$ tries

- **Less Server Storage**

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  - less than 4 KB

Differentially-Private Volume Hiding with dPRF vs DST

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Differentially-Private Volume Hiding with dPRF vs DST

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- Lossy with probability 2^{-64}
- Number of values of a label follows Zipf's distribution
  Average length 8
- To obtain 2^{-64}, $T = 5610$,
  - average download is 5618
- Volume-Hiding forced to download max length ≈ 84000
  15x improvement

Experiments

	Densest Subgraph Transform [KM19]				dprfMM				dpMM			
Input Multi-Map												
Number of Values (n)	2^{16}	2^{18}	2^{20}	2^{22}	2^{16}	2^{18}	2^{20}	2^{22}	2^{16}	2^{18}	2^{20}	2^{22}
Plaintext Raw Byte Size (MB)	1.05	4.19	16.78	67.11	1.05	4.19	16.78	67.11	1.05	4.19	16.78	67.11
EMM Storage												
Server (MB)	5.53	22.74	88.25	384.40	5.45	21.81	87.24	348.97	6.81	27.26	109.05	436.21
Client Stash (KB)	N/A	N/A	N/A	N/A	0.16	0.50	1.52	4.84	0.21	0.63	1.97	6.18
Query Communication												
Upload (Bytes)	16	16	16	16	16	16	16	16	36	36	36	36
Download (Bytes Per Result)	675.2	780.8	841.6	1008.0	64	64	64	64	64	64	64	64
CPU Costs												
Query (Client ms)	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01
Reply (Server ms Per Result)	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
Result (Client ms Per Result)	0.01	0.01	0.01	0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01	< 0.01

Table 2: Microbenchmarks for server and network costs comparing volume-hiding STE schemes. We denote n as the total number of values in the input multi-map. If ℓ is the maximum volume of any key, then the first two column constructions must download ℓ results. On the other hand, the number of results for dpMM will be significantly smaller than ℓ on average. The above results apply for any input multi-map structure, query distribution as well as any value ℓ. We denote milliseconds by ms.

Thank You

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